LESSON 10-8

Practice A

Combinations and Permutations

1. How many different ways can a bowl of cereal be made at a breakfast bar given the cereals and types of milk below?

Cereal	Milk
Granola	Skim
Crunchy Flakes	1%
Nut Crunch	2%

3 × 3 = 9

2. You must create a computer password consisting of 4 digits. Any digit can be repeated. How many different passwords can you create?

10 × 10 × 10 × 10 = 10,000

3. The choices for a pizza are shown below. How many different ways can the pizza be ordered?

Size	Crust	Shape
individual	thin crust	round
small	thick crust	rectangular
medium		
large		

4 × 2 × 2 = 16

Write C for combinations or P for permutations. Then answer the question.

4. Aaron has 6 autographed baseballs and room for 4 in his display case. How many arrangements can he make? — P; 360

5. Minnie has 4 different stamps. She needs 3 to mail a large letter. How many ways can she pick the 3 that will be used? — C; 4

6. How many ways can a group of 5 people be chosen from a class of 20? — C; 15,504

7. How many ways can 8 people line up for a picture? — P; 40,320

Answer each question.

8. Sergio is buying 3 different bagels. The bakery offers 7 types of bagels. How many different combinations of 3 bagels could Sergio buy? — 35

9. Four people are running in a relay race. How many different ways can the four people be arranged? — 24

10. A code consists of 3 letters. None of the letters can be repeated. How many different codes can there be? — 15,600

11. Ten business cards have been placed in a bowl at a restaurant. Four names are drawn to win a free lunch. How many different ways can the four names be drawn? — 210

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LESSON 10-8

Practice B

Combinations and Permutations

1. A code consists of 3 letters and then 3 digits. Any of the letters and numbers can be repeated. How many different codes are there? — 17,576,000

2. A restaurant is having a breakfast special. The choices are shown in the table. How many different breakfasts with one of each item are possible? — 24

Eggs	Meat	Bread	Juice
fried	bacon	biscuits	apple
scrambled	sausage	toast	orange
	ham		

3. A movie on DVD comes with different viewing options as shown in the table. How many different ways can the movie be watched? — 12

Audio	Commentary	Language
on	on	English
off	off	Spanish
		French

Write C for combinations or P for permutations. Then answer the question.

4. A coach must pick 5 players out of 30 to go on a trip. How many ways can the 5 players be chosen? — C; 142,506

5. Jenn has 5 types of flowers in her garden. How many ways can she make a bouquet consisting of 2 types of flowers? — C; 10

6. How many different ways can the letters in MUSIC be arranged? — P; 120

7. A grocery store carries 15 different types of cereals. Only 4 of the cereals can be displayed on the middle shelf. How many different ways can the 4 cereals be displayed? — P; 32,760

Answer each question.

8. A science fair awards prizes to the first, second and third place winners. There are 48 people entered in the science fair. How many ways can the winners be selected? — 103,776

9. A 3-digit computer password consists only of odd numbers that cannot be repeated. How many different 3-digit passwords are possible? — 60

10. In a lottery, 6 different numbers are selected from a set of 50 numbers. A winner can have the numbers in any order. How many sets of winning numbers are there? — 15,890,700

11. A band competition awards prizes to the top 3 schools. If 12 schools are entered, how many ways can 3 schools be chosen? — 220

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LESSON 10-8

Practice C

Combinations and Permutations

1. Andrea is buying a new computer. The computer is offered with 3 different types of keyboards, 4 sizes for memory, 3 different types of warranties, and 2 media packages. How many different ways can Andrea order a computer? — 72

2. Beginning with cars made in 1981, all vehicles must contain a 17-digit Vehicle Identification Number (VIN). The first 3 positions must be letters or numbers, positions 4 and 5 must be letters, positions 6 and 7 must be numbers, position 8 can be a letter or a number, position 9 is a number, position 10 is a letter or a number, and positions 11 through 17 must be numbers. All numbers and letters can be repeated. How many different VIN codes can there be? Write your answer in scientific notation; round the first factor to the hundredths. — 4.09×10^{20}

3. A baseball team is allowed to choose two colors as their team colors. Their choices of colors are: red, blue, orange, brown black, and silver. How many different possibilities are there for team colors? — 15

4. Sara is running errands. She must go to the bank, the grocery store, and the post office. How many ways can Sara complete all three errands? — 6

5. Four people are going to begin a game. They are deciding the order in which they will play. How many different ways are there for the 4 people to play? — 24

6. Twenty people have been summoned for jury duty. Twelve of the people will be chosen to serve on the jury. How many different juries can there be? — 125,970

7. In a 200 meter race, runners are assigned to different lanes. How many ways can 5 runners be assigned to 5 different lanes? — 120

8. A school is holding elections. The student with the greatest number of votes will be president and the student with the second greatest number of votes will be vice-president. If 20 people are in the election, how many different ways can the president and vice-president be chosen? — 380

9. Lily went to an animal shelter to choose 2 kittens as pets. The shelter had 38 kittens to choose from. How many ways did Lily have to choose her 2 new kittens? — 703

10. How many hands of 5 cards can be made from a standard deck of cards such that each hand contains exactly 1 red card and 4 black cards? — 388,700

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LESSON 10-8

Reteach

Combinations and Permutations

Permutations and combinations can be used when determining possible numbers of outcomes.

- A **permutation** is an arrangement of outcomes in which the *order matters.*
- A **combination** is a grouping of outcomes in which the *order does not matter.*

Tell whether each situation involves permutations or combinations.

A pizza place offers 3 toppings: olives, pepperoni, and mushrooms. How many 2-topping pizzas can Jack order?

An olive and pepperoni pizza is the same as a pepperoni and olive pizza. The order the toppings are placed on the pizza does not matter. This involves combinations.

Rosario has 5 awards. How many ways can she display 3 of them on her wall?

Changing the order of the awards changes the display. Because the order matters, this involves permutations.

The number of permutations of n things chosen r at a time is $_nP_r = \frac{n!}{(n-r)!}$.

Rosario has 5 awards. How many ways can she display 3 of them on her wall?

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

$n = 5$ and $r = 3$ because 3 out of 5 awards will be chosen.

$$= \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 5 \cdot 4 \cdot 3 = 60$$

There are 60 ways for Rosario to display the awards.

Tell whether each situation involves permutations or combinations.

1. How many different ways can 20 students line up for an assembly? — permutations

2. How many ways can a teacher choose 2 students from her class of 15 to distribute flyers to other classrooms? — combinations

Use the formula for permutations to answer each question.

3. Awards are given for first through fifth place. How many ways can the awards be given if 10 players compete? — 30,240

4. Hugo has 8 textbooks but only has room on his shelf for 4. How many ways can he arrange 4 textbooks on the shelf? — 1680

5. A 2-digit letter code is used to access a program. How many codes are possible if the same letter cannot be used twice? — 650

6. How many ways can 6 dogs be leashed to 6 leashes? — 720

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LESSON 10-8

Reteach

Combinations and Permutations (continued)

Because order does not matter in combinations, the total number of outcomes will be less than if the situation involved permutations.

The number of combinations of n things chosen r at a time is ${}_nC_r = \frac{n!}{r!(n-r)!}$.

A pizza place offers 10 toppings. Abel wants a pizza with 4 toppings. How many different 4-topping pizzas can Abel choose from?

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!}$$

($n = 10$ and $r = 4$ because 4 out of 10 toppings will be chosen.)

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(4 \cdot 3 \cdot 2 \cdot 1)(\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})}$$

$$= \frac{5040}{24}$$

$$= 210$$

Abel can choose from 210 pizzas.

Use the formula for combinations to answer each question.

7. A school is offering free dance tickets to 4 people from a class of 18. How many groups of 4 can be chosen? **3060**

8. An English exam consists of 7 possible questions. Students must choose 3 to answer. How many combinations of 3 questions are possible? **35**

9. From a group of 15 volunteers, 8 will be chosen for clean-up duty. How many groups of 8 are possible? **6435**

10. A smoothie shops offers 6 different fruits that can be blended together. How many 2-fruit smoothies can be ordered? **15**

The following situations involve either permutations or combinations. Answer each question.

11. A stationary catalog offers poster board in 5 different colors. Deonte must choose 2 colors for his history project. How many orders of 2 poster boards (of different color) are possible? **10**

12. How many ways can 4 people arrange themselves around a table with 12 seats? **11,880**

13. An advertisement for a grocery store states that when 6 cans of soup are purchased, $2 will be donated to a local charity. The store carries 11 flavors of soup. How many ways can 6 cans be chosen if each can is a different flavor? **462**

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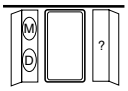
LESSON 10-8

Challenge

Circular and Ring Permutations

Three siblings, Aliyah, Bentley, and Chris, go out for an evening with their parents. Use permutations to answer these questions.

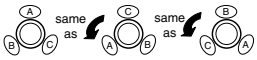
1. The family first goes out for dinner. A large booth is available. Mom and Dad sit on one side and the kids sit on the other side. How many different arrangements of the siblings are there?



${}_3P_3 = 3! = 6$

2. After dinner, the family goes to an ice cream shop for dessert. The shop has little round tables, so Mom and Dad sit at one table and the siblings sit at another. How many different arrangements of the siblings are there?

Note: When objects are arranged in a circle, any arrangement that is merely a rotation of another is considered to be the same. For example, all three of the arrangements at right are the same.



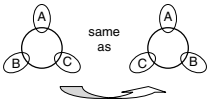
$\frac{6}{3} = 2$

3. In Lesson 10-8, you learned that the number of *linear* permutations (arrangements in a line) of n things chosen r at a time is represented by ${}_nP_r$. If those r items are arranged in a circle, what expression represents the number of *circular* permutations?

$\frac{{}_nP_r}{r}$

4. The family then goes to the mall, where Mom buys a bracelet (without a clasp) and three charms with her children's names on them. How many different ways can Mom hang the charms on her bracelet?

Note: When objects are arranged in a ring that can be flipped upside down, any arrangement that is a mirror reflection of another is considered to be the same arrangement. But, remember that a ring is also a circle!



$\frac{6}{2 \cdot 3} = 1$

5. If the number of linear permutations of n things chosen r at a time is represented by ${}_nP_r$, what expression represents the number of *ring* permutations of those r items?

$\frac{{}_nP_r}{2 \cdot r}$

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LESSON 10-8

Problem Solving

Combinations and Permutations

Write the correct answer.

1. In 1963, the United States Postal Service started using ZIP (Zone Improvement Plan) codes. Each ZIP code is a sequence of five digits. How many different U.S. ZIP codes are possible?

100,000

2. In the early 1970's, Canada Post started using six-character postal codes. Each postal code uses three letters and three digits in an alternating pattern. How many different Canadian postal codes are possible?

17,576,000

3. A band's new album contains 12 songs. The record company decides to promote the album at a music festival by giving away CD samplers that contain 3 songs from the album.

   a. Does this situation involve combinations or permutations? Explain.

   permutations; the tracks on a CD are in a specific order.

   b. How many different ways can the band select songs for the sampler?

   1320

4. A 12-person jury is being selected for a court trial. After lengthy interviews, the attorney's have narrowed the candidates down to 14 people that they both think are fair and unbiased.

   a. Does this situation involve combinations or permutations? Explain.

   combinations; the members of a jury have no particular order.

   b. How many different ways can the attorneys select the jury?

   91

Select the best answer.

5. Myra works Monday to Thursday as a waitress. She has 6 short-sleeve shirts that are appropriate for her to wear as part of her uniform. If she doesn't wear the same shirt twice, how many different ways can she wear her shirts during a 4-day work week?

   A 15 **C** 360
   B 24 **D** 720

6. At Cold Marble Ice Cream, you create your own ice cream flavor by choosing from a list of 10 "add-ins" that can be mixed into vanilla ice-cream. If you can pick one add-in, any combination of two add-ins, or any combination of three add-ins, how many different flavors are possible?

   F 120 **H** 210
   G 175 **J** 820

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LESSON 10-8

Reading Strategies

Use a Concept Map

This concept map will help you understand combinations and permutations.

Definition A *combination* is a grouping of outcomes in which the order does not matter.	**Formula** The number of *combinations* of n things chosen r at a time is found using ${}_nC_r = \frac{n!}{r!(n-r)!}$.	**Example** How many pairs can be made from a group of 6 people? (Jin and Tom are the same pair as Tom and Jin) ${}_6C_2 = \frac{6!}{2!(6-2)!} = \frac{720}{2(24)} = 15$
Combinations		
Permutations		
Definition A *permutation* is an arrangement of outcomes in which the order does matter.	**Formula** The number of *permutations* of n things chosen r at a time is found using ${}_nP_r = \frac{n!}{(n-r)!}$.	**Example** 6 people are in a contest. How many ways can 1st and 2nd place be awarded? (Jin first and Tom second is different than Tom first and Jin second) ${}_6P_2 = \frac{6!}{(6-2)!} = \frac{720}{24} = 30$

Tell whether each situation involves combinations or permutations. Then find the number of possible outcomes.

1. For her birthday, Marianna may choose 3 DVDs from the 5 that her mother picked out. How many ways can Marianna choose her DVDs?

   combinations; 10

2. A box has 8 hats. A group of 5 children playing dress up each choose a hat from the box. How many ways can the children choose their hats?

   permutations; 6720

3. Six students tried out for 3 different roles in a school play. How many ways could the director fill the roles?

   permutations; 120

4. Fiona is going to order bunches of balloons for a party. In each bunch she wants 4 different colored balloons. The store offers 7 colors. How many ways could Fiona make one bunch?

   combinations; 35

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