LESSON 11-1

## Practice A

### Geometric Sequences

**Find the common ratio of each geometric sequence. Then find the next three terms in each geometric sequence.**

**1.** 1, 4, 16, 64, …

common ratio: 4

256, 1024, 4096

**2.** 10, 100, 1000, 10,000, …

common ratio: 10

100,000, 1,000,000, 10,000,000

**3.** 128, 64, 32, 16, …

common ratio: $\frac{1}{2}$

8, 4, 2

**4.** 4, −20, 100, −500, …

common ratio: −5

2500, −12,500, 62,500

**5.** The first term of a geometric sequence is 2 and the common ratio is 4. Find the 6th term.

2048

**6.** The first term of a geometric sequence is −3 and the common ratio is 2. Find the 8th term.

−384

**7.** The first term of a geometric sequence is 7 and the common ratio is −2. Find the 9th term.

1792

**8.** What is the 5th term of the geometric sequence 9, 27, 81, 243, …?

common ratio ($r$): 3

first term ($a_1$): 9

5th term: 729

**9.** What is the 13th term of the geometric sequence −2, 4, −8, 16, …?

common ratio ($r$): −2

first term ($a_1$): −2

13th term: −8192

**10.** Martin got a job at a starting pay of $8.00 per hour. His boss told him that if he works hard he can get a raise each year. The table shows Martin's wage for the first few years. Find Martin's hourly wage after 6 years. Round to the nearest cent.

common ratio ($r$): 1.2

first term ($a_1$): 8

6th term: $19.91

| Year | Hourly Wage ($) |
|---|---|
| 1 | $8.00 |
| 2 | $9.60 |
| 3 | $11.52 |

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LESSON 11-1

## Practice B

### Geometric Sequences

**Find the next three terms in each geometric sequence.**

**1.** −5, −10, −20, −40, …

−80, −160, −320

**2.** 7, 56, 448, 3584…

28,672, 229,376, 1,835,008

**3.** −10, 40, −160, 640, …

−2560, 10,240, −40,960

**4.** 40, 10, $\frac{5}{2}$, $\frac{5}{8}$, …

$\frac{5}{32}, \frac{5}{128}, \frac{5}{512}$

**5.** The first term of a geometric sequence is 6 and the common ratio is −8. Find the 7th term.

1,572,864

**6.** The first term of a geometric sequence is −3 and the common ratio is $\frac{1}{2}$. Find the 6th term.

$-\frac{3}{32}$

**7.** The first term of a geometric sequence is −0.25 and the common ratio is −3. Find the 10th term.

4920.75

**8.** What is the 12th term of the geometric sequence −4, −12, −36, …?

−708,588

**9.** What is the 10th term of the geometric sequence 2, −6, 18, …?

−39,366

**10.** What is the 6th term of the geometric sequence 50, 10, 2, …?

0.016

**11.** A shoe store is discounting shoes each month. A pair of shoes cost $80. The table shows the discount prices for several months. Find the cost of the shoes after 8 months. Round your answer to the nearest cent.

| Month | Price |
|---|---|
| 1 | $80.00 |
| 2 | $72.00 |
| 3 | $64.80 |

$38.26

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LESSON 11-1

## Practice C

### Geometric Sequences

**Find the next three terms in each geometric sequence.**

**1.** 0.1, 0.5, 2.5, 12.5…

62.5, 312.5, 1562.5

**2.** −3.6, −5.4, −8.1, …

−12.15, −18.225, −27.3375

**3.** 1, $-\frac{1}{10}, \frac{1}{100}, -\frac{1}{1000}$, …

$\frac{1}{10{,}000}, -\frac{1}{100{,}000}, \frac{1}{1{,}000{,}000}$

**4.** $-\frac{40}{3}$, −10, $-\frac{15}{2}$, …

$-\frac{45}{8}, -\frac{135}{32}, -\frac{405}{128}$

**5.** The first term of a geometric sequence is 0.5 and the common ratio is 5. Find the 15th term.

3,051,757,813

**6.** The first term of a geometric sequence is −10,000 and the common ratio is $-\frac{1}{2}$. Find the 6th term.

312.5

**7.** The first term of a geometric sequence is $\frac{3}{4}$ and the common ratio is 4. Find the 8th term.

12,228

**8.** What is the 5th term of the geometric sequence 10, 8, 6.4, …?

4.096

**9.** What is the 10th term of the geometric sequence −20, −10, −5, …?

$-\frac{5}{128}$

**10.** What is the 6th term of the geometric sequence $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}$, …?

$-\frac{1}{729}$

**Raheem was offered two different jobs as a webmaster. Each job had different starting annual salaries and different increases each year. The table shows the salaries for the first few years. Use this information to answer questions 11 −13. Round all answers to the nearest dollar.**

| Year | Job A | Job B |
|---|---|---|
| 1 | $30,000.00 | $24,000.00 |
| 2 | $30,660.00 | $25,080.00 |
| 3 | $31,334.52 | $26,209.00 |

**11.** What is the annual salary for Job A in the 5th year?

$32,728

**12.** What is the annual salary for Job B in the 5th year?

$28,620

**13.** At what year will Job B pay a higher annual salary than Job A?

12th year

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LESSON 11-1

## Reteach

### Geometric Sequences

In a **geometric sequence**, each term is *multiplied* by the same number to get to the next term. This number is called the **common ratio**.

3, 12, 48, 192 (×4, ×4, ×4) — The common ratio is 4.

**Determine if the sequence 2, 6, 18, 54, … is a geometric sequence.**

Divide each term by the term before it.

$\frac{54}{18} = 3 \quad \frac{18}{6} = 3 \quad \frac{6}{2} = 3$

This is a geometric sequence; 3 is the common ratio.

**Determine if the sequence 5, 10, 15, 20, … is a geometric sequence.**

Divide each term by the term before it.

$\frac{20}{15} = \frac{4}{3} \quad \frac{15}{10} = \frac{3}{2} \quad \frac{10}{5} = 2$

This is not a geometric sequence; there is no common ratio.

**Find the next three terms in the geometric sequence 1, 4, 16, 64, ….**

**Step 1**: Find the common ratio.

$\frac{64}{16} = 4 \quad \frac{16}{4} = 4 \quad \frac{4}{1} = 4$

**Step 2**: Continue to multiply by the common ratio.

$64 \times 4 = 256 \quad 256 \times 4 = 1024 \quad 1024 \times 4 = 4096$

The next three terms are 256, 1024, and 4096.

**Determine if each sequence is a geometric sequence. Explain.**

**1.** 2, 4, 6, 8, … no; there is no common ratio.

**2.** −4, 8, −16, 32, … yes; the common ratio is −2.

**3.** 32, 16, 8, 4,… yes; the common ratio is $\frac{1}{2}$.

**Find the common ratio in each geometric sequence below. Then find the next three terms.**

**4.** 1, 5, 25, 125, …

5; 625, 3125, 15,625

**5.** −6, 12, −24, 48, …

−2; −96, 192, −384

**6.** 4, 6, 9, 13.5, …

1.5; 20.25, 30.375, 45.5625

**7.** $\frac{1}{4}, \frac{1}{2}$, 1, 2, …

2; 4, 8, 16

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LESSON 11-1

## Reteach

### *Geometric Sequences (continued)*

There are two ways to find a given term of a geometric sequence.

**Find the 8th term in the geometric sequence 5, 10, 20, 40, ….**

**Method 1**: Extend the sequence to the 8th term.

**Step 1**: Find the common ratio.

$\frac{40}{20} = 2 \quad \frac{20}{10} = 2 \quad \frac{10}{5} = 2$

The common ratio is 2.

**Step 2**: Continue to multiply each term by 2.

5, 10, 20, 40, 80, 160, 320, **640,** …

Stop at the 8th term.

The 8th term is 640.

**Method 2**: Use a formula to find the 8th term.

Look at Method 1. The first term, 5, was multiplied by 2 seven times to get to the eighth term.

8th term $= 5(2)(2)(2)(2)(2)(2)(2) = 5(2)^7$

Written as a formula, this would be:

$$a_n = a_1\, r^{n-1},$$

where $n$ is the number of terms and $r$ is the common ratio.

To find the 8th term of the sequence, use $n = 8$ and $r = 2$.

$a_n = a_1\, r^{n-1}$
$a_8 = 5(2)^{8-1}$
$a_8 = 5(2)^7$
$a_8 = 5(128)$
$a_8 = 640$

The 8th term is 640.

**Find the indicated term.**

**8.** $a_1 = 7$, $r = -2$; 10th term

$a_{10} = 7(-2)^{10-1}$

−3584

**9.** $a_1 = -4$, $r = 3$; 8th term

$a_8 = -4(3)^{8-1}$

−8748

**10.** The first term of a geometric sequence is 2, and the common ratio is 3. What is the 7th term?

1458

**11.** The first term of a geometric sequence is −3, and the common ratio is −2. What is the 9th term?

−768

**12.** Find the 12th term in the geometric sequence 5, −15, 45, −135, ….

−885,735

**13.** Find the 8th term in the geometric sequence 243, 81, 27, 9, ….

$\frac{1}{9}$

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LESSON 11-1

## Challenge

### *Series*

A **sequence** is a list of numbers that often forms a pattern. You have worked with arithmetic sequences and geometric sequences.

A **series** is the sum of the terms of a sequence. You can either find a *partial sum* of a few terms or determine what happens when you sum infinitely many terms. (You'll look at infinite series in Algebra 2.)

The Greek capital letter sigma $(\sum)$ is often used to represent summation in mathematics. Numbers above and below sigma tell you which terms to sum.

For example, $\sum_{n=1}^{5}$ represents the partial sum of the 1st to 5th terms.

**Find the partial sums of these arithmetic series.**

**1.** $\sum_{n=1}^{5} 4 + (n-1)(3) = \underbrace{4}_{n=1} + \underbrace{7}_{n=2} + \underbrace{10}_{n=3} + \underbrace{13}_{n=4} + \underbrace{16}_{n=5} =$ 50

**2.** $\sum_{n=1}^{8} -10 + (n-1)(2) = \underbrace{-10}_{n=1} + \underbrace{-8}_{n=2} + \underbrace{-6}_{n=3} + \underbrace{-4}_{n=4} + \underbrace{-2}_{n=5} + \underbrace{0}_{n=6} + \underbrace{2}_{n=7} + \underbrace{4}_{n=8}$ = −24

**3.** $\sum_{n=1}^{6} 23 + (n-1)(-5) =$ 23 + 18 + 13 + 8 + 3 + (−2) = 63

**4.** $\sum_{n=1}^{4} \frac{1}{2} + (n-1)\left(\frac{1}{8}\right) =$ $\frac{1}{2} + \frac{5}{8} + \frac{3}{4} + \frac{7}{8}$ = $\frac{11}{4}$ or $2\frac{3}{4}$

**5.** $\sum_{n=5}^{10} -7 + (n-1)(-1.2) =$ −11.8 + (−13) + (−14.2) + (−15.4) + (−16.6) + (−17.8) = −88.8

Caution!

**Find the partial sums of these geometric series.**

**6.** $\sum_{n=1}^{5} 4(3)^{n-1} = \underbrace{4}_{n=1} + \underbrace{12}_{n=2} + \underbrace{36}_{n=3} + \underbrace{108}_{n=4} + \underbrace{324}_{n=5} =$ 484

**7.** $\sum_{n=1}^{8} -10(2)^{n-1} = \underbrace{-10}_{n=1} + \underbrace{-20}_{n=2} + \underbrace{-40}_{n=3} + \underbrace{-80}_{n=4} + \underbrace{-160}_{n=5} + \underbrace{-320}_{n=6} + \underbrace{-640}_{n=7} + \underbrace{-1280}_{n=8}$ = −2550

**8.** $\sum_{n=1}^{6} 3(-2)^{n-1} =$ 3 + (−6) + 12 + (−24) + 48 + (−96) = −63

**9.** $\sum_{n=1}^{4} 16\left(\frac{1}{4}\right)^{n-1} =$ 16 + 4 + 1 + $\frac{1}{4}$ = $\frac{85}{4}$ or $21\frac{1}{4}$

**10.** $\sum_{n=5}^{10} -6\left(-\frac{2}{3}\right)^{n-1} = \underbrace{-\frac{32}{27}}_{n=5} + \underbrace{\frac{64}{81}}_{n=6} + \underbrace{\left(-\frac{128}{243}\right)}_{n=7} + \underbrace{\frac{256}{729}}_{n=8} + \underbrace{\left(-\frac{512}{2187}\right)}_{n=9} + \underbrace{\frac{1024}{6561}}_{n=10} =$ $\frac{-4256}{6561} \approx -0.649$

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LESSON 11-1

## Problem Solving

### *Geometric Sequences*

**Write the correct answer.**

**1.** A ball is dropped from 400 feet. The table shows the height of each bounce.

| Bounce | Height (ft) |
|---|---|
| 1 | 280 |
| 2 | 196 |
| 3 | 137.2 |

Find the height of the ball on the 6th bounce. Round your answer to the nearest tenth of a foot.

47.1 feet

**2.** A plant starts with 1 branch. Every year, each branch becomes 3 branches. A sketch of the plant for the first 3 years is shown. How many branches will the plant have in year 10?




19,683 branches

How many branches would the plant have in year 10 if the plant had 5 branches the first year? (Each branch still becomes 3 branches every year.)

98,415 branches

**3.** Jeanette started selling bagels to offices in her area. Her sales for the first 3 months are shown in the table.

| Month | Sales ($) |
|---|---|
| 1 | $200.00 |
| 2 | $230.00 |
| 3 | $264.50 |

If this trend continues, find the amount of Jeanette's sales in Month 8.

$532

**The table shows the number of houses in a new subdivision. Use the table to answer questions 4–7. Select the best answer.**

| Month | Houses |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 12 |
| 4 | 24 |

**4.** The number of houses forms a geometric sequence. What is $r$?

**A** 0.5 **C** 3
**B** 2 **D** 6

**5.** Assuming that the trend continues, how many houses would be in the subdivision in Month 6?

**F** 36 **H** 60
**G** 48 **J** 96

**6.** Management decides the subdivision is complete when the number of houses reaches 48. When will this happen?

**A** Month 5 **C** Month 7
**B** Month 6 **D** Month 8

**7.** Suppose the number of houses tripled every month. How many more houses would be in the subdivision in Month 4? (The number of houses in Month 1 is still 3.)

**F** 48 **H** 72
**G** 57 **J** 81

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LESSON 11-1

## Reading Strategies

### *Understanding Concepts*

Study the information below to help you understand geometric sequences and the formula used to find terms.

In a *geometric sequence* each number is the same multiple of the number before it.

This is the formula used to find the value of the $n$th term, $a_n$, in a geometric sequence.

The first term, $a_1$, is 1.

1, −4, 16, −64, 256...

×−4 ×−4 ×−4 ×−4

$a_n = a_1 r^{n-1}$

Using a calculator, the 10th term is $1(-4)^{10-1} = 1(-4)^9 = -262,144$.

The *common ratio*, $r$, is −4.

To find the 10th term in this sequence, we'd have $a_{10} = 1(-4)^{10-1}$.

**Complete the following about the geometric sequence 2, 6, 18, 54, ... .**

**1.** What is the first term, $a_1$? 2

**2.** What is the common ratio, $r$? 3

**3.** Describe how you found the common ratio in problem 2.

Possible answer: I divided each term by the previous term.

**4.** What is the next term? 162

**5.** Use the formula to find the 12th term. 354,294

**Complete the following.**

**6.** Is 6, 9, 12, 15, ... a geometric sequence? Explain.

no; the ratios between terms (1.5, $1.\overline{3}$, and 1.25) are different.

**7.** Find the 8th term in the geometric sequence 5, −10, 20, −40, ... . −640

**8.** The first term of a geometric sequence is 14 and the common ratio is 6. Find the 5th term. 18,144

**9.** Find the 9th term in the geometric sequence 256, 128, 64, 32, ... . 1

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