LESSON 11-6

Practice A

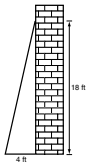
Radical Expressions

Complete the steps to simplify each expression.

1. $\sqrt{\frac{32}{8}} = \sqrt{4} =$ 2
2. $\sqrt{72 - 8} = \sqrt{64} =$ 8
3. $\sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} =$ 13
4. $\sqrt{(b-4)^2} =$ $|b - 4|$
5. $\sqrt{10^2 - 19} =$ 9
6. $\sqrt{x^2 + 6x + 9} =$ $|x + 3|$

Simplify. All variables represent nonnegative numbers.

7. $\sqrt{72} = \sqrt{36 \cdot 2}$
   $= \sqrt{36} \cdot \sqrt{2}$
   $= 6 \cdot \sqrt{2}$
8. $\sqrt{300} = \sqrt{100 \cdot 3}$
   $= \sqrt{100} \cdot \sqrt{3}$
   $= 10\sqrt{3}$
9. $\sqrt{b^2 c^2} = \sqrt{b^2 \cdot c^2}$
   $= \sqrt{b^2} \cdot \sqrt{c^2}$
   $= bc$
10. $\sqrt{500} = \sqrt{5 \cdot 100}$
    $10\sqrt{5}$
11. $\sqrt{90} = \sqrt{9 \cdot 10}$
    $3\sqrt{10}$
12. $\sqrt{98xy^2}$
    $7y\sqrt{2x}$
13. $\sqrt{\frac{1}{100}} = \frac{\sqrt{1}}{\sqrt{100}}$
    $= \frac{1}{10}$
14. $\sqrt{\frac{9y^2}{16x^4}} = \frac{\sqrt{9y^2}}{\sqrt{16x^4}}$
    $= \frac{3y}{4x^2}$
15. $\sqrt{\frac{21b^2}{25c^4}} = \frac{\sqrt{21b^2}}{\sqrt{25c^4}}$
    $= \frac{b\sqrt{21}}{5c^2}$
16. $\sqrt{\frac{x^3}{y^2}}$
    $\frac{x\sqrt{x}}{y}$
17. $\sqrt{\frac{4x^4}{100x^2}}$
    $\frac{x}{5}$
18. $\sqrt{\frac{10b^3}{16b^2}}$
    $\frac{\sqrt{10b}}{4}$
19. $\sqrt{\frac{32}{9}}$
    $\frac{4\sqrt{2}}{3}$
20. $\sqrt{\frac{121x^5}{9}}$
    $\frac{11x^2\sqrt{x}}{3}$
21. $\sqrt{\frac{288x^9}{25}}$
    $\frac{12x^4\sqrt{2x}}{5}$
22. A ladder is propped up against a house. The top of the ladder rests 18 ft above the ground and the base of the ladder is 4 ft from the building. Write the length of the ladder as a radical expression in simplest form. Then estimate the length to the nearest hundredth of a foot.



$2\sqrt{85}$ ft; 18.44 ft

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LESSON 11-6

Practice B

Radical Expressions

Simplify each expression.

1. $\sqrt{225} =$ 15
2. $\sqrt{\frac{75}{3}} = \sqrt{25} =$ 5
3. $\sqrt{7^2 + 24^2} =$ 25
4. $\sqrt{(x+8)^2} =$ $|x + 8|$
5. $\sqrt{\frac{4}{100}} = \frac{1}{5}$
6. $\sqrt{x^2 + 8x + 16} =$ $|x + 4|$

Simplify. All variables represent nonnegative numbers.

7. $\sqrt{32}$
   $4\sqrt{2}$
8. $\sqrt{28}$
   $2\sqrt{7}$
9. $\sqrt{x^4 y^3}$
   $x^2y\sqrt{y}$
10. $\sqrt{147}$
    $7\sqrt{3}$
11. $\sqrt{45}$
    $3\sqrt{5}$
12. $\sqrt{36x^4 y^5}$
    $6x^2y^2\sqrt{y}$
13. $\sqrt{\frac{7}{25}}$
    $\frac{\sqrt{7}}{5}$
14. $\sqrt{\frac{3b^2}{27b^4}}$
    $\frac{1}{3b}$
15. $\sqrt{\frac{m^3}{121n^4}}$
    $\frac{m\sqrt{m}}{11n^2}$
16. $\sqrt{\frac{10b^4}{2b^3}}$
    $\sqrt{5b}$
17. $\sqrt{\frac{9y^6}{36y^2}}$
    $\frac{y^2}{2}$
18. $\sqrt{\frac{40m^3}{10n^4}}$
    $\frac{2m\sqrt{m}}{n^2}$
19. $\sqrt{\frac{128}{25}}$
    $\frac{8\sqrt{2}}{5}$
20. $\sqrt{\frac{4}{81x^8}}$
    $\frac{2}{9x^4}$
21. $\sqrt{\frac{250q^{10}}{5q^4}}$
    $5q^3\sqrt{2}$
22. Two hikers leave a ranger station at noon. Tom heads due south at 5 mi/h and Kyle heads due east at 3 mi/h. How far apart are the hikers at 4 PM? Give your answer as a radical expression in simplest form. Then estimate the distance to the nearest tenth of a mile.

$4\sqrt{34}$ mi; 23.3 mi

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LESSON 11-6

Practice C

Radical Expressions

Simplify each expression.

1. $\sqrt{900}$
   30
2. $\sqrt{\frac{242}{2}}$
   11
3. $\sqrt{(n-10)^2}$
   $|n - 10|$

Simplify. All variables represent nonnegative numbers.

4. $\sqrt{80}$
   $4\sqrt{5}$
5. $\sqrt{108}$
   $6\sqrt{3}$
6. $\sqrt{450}$
   $15\sqrt{2}$
7. $\sqrt{s^3t^2}$
   $st\sqrt{s}$
8. $\sqrt{49x^2y^4}$
   $7xy^2$
9. $\sqrt{98x^2y}$
   $7x\sqrt{2y}$
10. $\sqrt{\frac{21}{25}}$
    $\frac{\sqrt{21}}{5}$
11. $\sqrt{\frac{45}{16}}$
    $\frac{3\sqrt{5}}{4}$
12. $\sqrt{\frac{500}{64}}$
    $\frac{5\sqrt{5}}{4}$
13. $\sqrt{\frac{150}{s^2}}$
    $\frac{5\sqrt{6}}{s}$
14. $\sqrt{\frac{25b^4}{196c^6}}$
    $\frac{5b^2}{14c^3}$
15. $\sqrt{\frac{18x^2}{25y^4}}$
    $\frac{3x\sqrt{2}}{5y^2}$
16. $\sqrt{\frac{50x^2y^3}{49x^4y}}$
    $\frac{5y\sqrt{2}}{7x}$
17. $\sqrt{\frac{f^2g^2}{h^2g}}$
    $\frac{f\sqrt{g}}{h}$
18. $-\sqrt{\frac{b^2}{100b}}$
    $-\frac{\sqrt{b}}{10}$
19. $-3\sqrt{9n^4}$
    $-9n^2$
20. $-\frac{1}{5}\sqrt{\frac{405}{320}}$
    $-\frac{9}{40}$
21. $2x\sqrt{\frac{162x^3}{16x}}$
    $\frac{9x^2\sqrt{2}}{2}$
22. Three cruise ships left a harbor at the same time. The *Princess Wilma* traveled due north at 18 mi/h, the *Prince Theodore* traveled due west at 14 mi/h, and the *King Frank* traveled due east at 12 mi/h. How far is the *Princess Wilma* from the other two ships after two hours? Give each answer as a radical expression in simplest form. Then estimate each answer to the nearest tenth of a mile.

Prince Theodore: $4\sqrt{130}$ mi, ≈45.6 mi; ***King Frank***: $12\sqrt{13}$ mi, ≈43.3 mi

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LESSON 11-6

Reteach

Radical Expressions

A **radical expression** is an expression that contains a radical sign.

$\sqrt{14x}$ — The expression under the radical sign is the **radicand**.

A radical expression is in simplest form if:

- the radicand has no perfect square factors other than 1
- the radicand has no fractions
- there are no square roots in the denominator

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$; where $a \geq 0$ and $b \geq 0$

Simplify $\sqrt{50}$.

$\sqrt{50} = \sqrt{25 \cdot 2}$ *Write the radicand as a product.*

$= \sqrt{25} \cdot \sqrt{2}$ *Use Product Property of Square Roots*

$= 5\sqrt{2}$ *Simplify.*

Simplify $\sqrt{x^2y}$.

$\sqrt{x^2y} = \sqrt{x^2}\sqrt{y}$

$= x\sqrt{y}$

Quotient Property of Square Roots

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$; where $a \geq 0$ and $b > 0$

Simplify $\sqrt{\frac{3}{49}}$.

$\sqrt{\frac{3}{49}} = \frac{\sqrt{3}}{\sqrt{49}}$ *Use Quotient Property of Square Roots.*

$= \frac{\sqrt{3}}{7}$ *Simplify.*

Simplify $\sqrt{\frac{x^6}{16}}$.

$\sqrt{\frac{x^6}{16}} = \frac{\sqrt{x^6}}{\sqrt{16}}$

$= \frac{x^3}{4}$

Simplify. All variables represent nonnegative numbers.

1. $\sqrt{20}$
   $2\sqrt{5}$
2. $\sqrt{300}$
   $10\sqrt{3}$
3. $\sqrt{54x^4}$
   $3x^2\sqrt{6}$
4. $\sqrt{\frac{7}{81}}$
   $\frac{\sqrt{7}}{9}$
5. $\sqrt{\frac{10}{9}}$
   $\frac{\sqrt{10}}{3}$
6. $\sqrt{\frac{9x^8}{25y^6}}$
   $\frac{3x^4}{5y^3}$

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LESSON 11-6

Reteach

Radical Expressions (continued)

The Product and Quotient Properties can be used together to simplify radical expressions.

Simplify $\sqrt{\frac{28}{25}}$.

$\sqrt{\frac{28}{25}} = \frac{\sqrt{28}}{\sqrt{25}}$ *Quotient Property*

$= \frac{\sqrt{4 \cdot 7}}{\sqrt{25}}$ *Write 28 as 4 × 7.*

$= \frac{\sqrt{4} \cdot \sqrt{7}}{\sqrt{25}}$ *Product Property*

$= \frac{2\sqrt{7}}{5}$ *Simplify.*

Simplify $\sqrt{\frac{9x^3}{49}}$. All variables represent nonnegative numbers.

$\sqrt{\frac{9x^3}{49}} = \frac{\sqrt{9x^3}}{\sqrt{49}}$ *Quotient Property*

$= \frac{\sqrt{9 \cdot x^2 \cdot x}}{\sqrt{49}}$ *Write x^3 as $x^2(x)$.*

$= \frac{\sqrt{9}\sqrt{x^2}\sqrt{x}}{\sqrt{49}}$ *Product Property*

$= \frac{3x\sqrt{x}}{7}$ *Simplify.*

Simplify by filling in the blanks below. All variables represent nonnegative numbers.

7. $\sqrt{\frac{75}{4}} = \frac{\sqrt{\boxed{75}}}{\sqrt{\boxed{4}}}$

$\frac{\sqrt{\boxed{25}}\sqrt{\boxed{3}}}{\sqrt{\boxed{4}}}$

$\frac{\boxed{5}\sqrt{\boxed{3}}}{\boxed{2}}$

8. $\sqrt{\frac{288}{25}} = \frac{\sqrt{\boxed{288}}}{\sqrt{\boxed{25}}}$

$\frac{\sqrt{\boxed{144}}\sqrt{\boxed{2}}}{\sqrt{\boxed{25}}}$

$\frac{\boxed{12}\sqrt{\boxed{2}}}{\boxed{5}}$

Simplify. All variables represent nonnegative numbers.

9. $\sqrt{\frac{8}{81}}$ $\frac{2\sqrt{2}}{9}$

10. $\sqrt{\frac{18}{49}}$ $\frac{3\sqrt{2}}{7}$

11. $\sqrt{\frac{500}{36}}$ $\frac{5\sqrt{5}}{3}$

12. $\sqrt{\frac{242x^2}{9}}$ $\frac{11x\sqrt{2}}{3}$

13. $\sqrt{\frac{m^7}{16n^2}}$ $\frac{m^3\sqrt{m}}{4n}$

14. $\sqrt{\frac{200x^2}{49y^2}}$ $\frac{10x\sqrt{2}}{7y}$

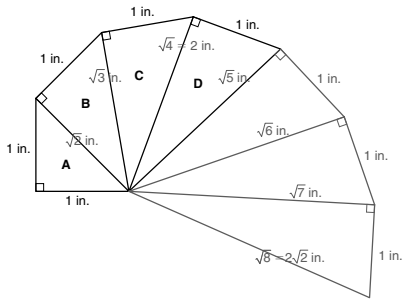
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LESSON 11-6

Challenge

The Square-Root Spiral

The diagram below is the beginning of a square-root spiral. It begins with an isosceles right triangle (A) that has two 1-inch legs. Then another right triangle (B) with a 1-inch leg is constructed on the hypotenuse, and then another (C), and another (D), and so on.



1. Use the Pythagorean Theorem to calculate the length of the hypotenuse of each triangle above. Give each answer as a radical expression. Label the lengths on the diagram as well.

a. Triangle A $\sqrt{2}$ in.

b. Triangle B $\sqrt{3}$ in.

c. Triangle C $\sqrt{4} = 2$ in.

d. Triangle D $\sqrt{5}$ in.

2. Describe any patterns you see in the lengths of the hypotenuses.

The length of each hypotenuse is a square root expression in which the radicand increases by 1 from the previous hypotenuse.

3. Use a ruler, protractor, and/or compass to draw the next three right triangles in the spiral. Extend the pattern in the lengths of the hypotenuses to label the sides of each new triangle.

4. How could you draw a segment that represents $\sqrt{13}$?

Continue the spiral until you have 12 triangles; the length of the hypotenuse of the 12th triangle will be $\sqrt{13}$ in.

5. In general, what is the length of the hypotenuse of the nth triangle in the spiral?

$\sqrt{n+1}$ in.

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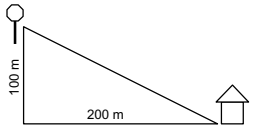
LESSON 11-6

Problem Solving

Radical Expressions

Write the correct answer.

1. Annalise walks 200 meters from her house, turns a corner, and then walks another 100 meters to the bus stop. She wants to know how much shorter her walk would be if she cut across the field. Find the distance across the field from Annalise's house to the bus stop. Give your answer as a radical expression in simplest form. Then find the difference in the two routes to the nearest meter.



224 m; 76 m

2. A construction worker drops a nail from some scaffolding 192 feet from the ground. The nail is in freefall. The time t in seconds for an object in freefall to reach the ground is $t = \sqrt{\frac{d}{16}}$, where d is the distance in feet that it falls. The speed v in feet per second of an object in freefall is modeled by $v = 8\sqrt{d}$. Determine how long it takes the nail to reach the ground. Also find the speed the nail is traveling at 192 feet. Give your answers as radical expressions in simplest form. Then estimate the time to the nearest second and the speed to the nearest foot per second.

time: $2\sqrt{3}$ s, 3 s

speed: $64\sqrt{3}$ ft/s, 111 ft/s

Police officers determine the speed a car was traveling when the driver slammed on the brakes by measuring the length of skid marks left by the tires. On dry concrete, $f(x) = \sqrt{24x}$ gives the speed in mi/h when the length of the skid marks is x feet. The graph shows lengths of skid marks from several cars. Select the best answer.

3. Which shows the speed of car A in simplest radical form?

A $\sqrt{3360}$ mi/h **C** $4\sqrt{210}$ mi/h

B $2\sqrt{840}$ mi/h **D** $16\sqrt{210}$ mi/h

4. Which shows the speed of car B in simplest radical form?

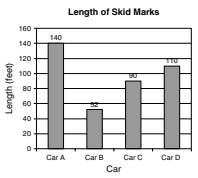
F $2\sqrt{78}$ mi/h **H** $8\sqrt{78}$ mi/h

G $4\sqrt{78}$ mi/h **J** $16\sqrt{78}$ mi/h

5. Which shows the speed of car C in simplest radical form?

A $2\sqrt{540}$ mi/h **C** $12\sqrt{15}$ mi/h

B $4\sqrt{135}$ mi/h **D** $16\sqrt{135}$ mi/h



6. The driver of car D claims his skids marks were really only 60% as long as officers claim they were. If the driver is telling the truth, what was his speed in simplest radical form?

F $\sqrt{66}$ mi/h **H** $2\sqrt{396}$ mi/h

G $\sqrt{1584}$ mi/h **J** $12\sqrt{11}$ mi/h

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LESSON 11-6

Reading Strategies

Follow a Procedure

When simplifying radical expressions, there may be several steps that need to be taken. Look at the example below.

Simplify $\sqrt{\frac{12x^6}{25y^2}}$. All variables represent nonnegative numbers.

$\sqrt{\frac{12x^6}{25y^2}} = \sqrt{\frac{12x^6}{25y^2}}$ — 1 — Use the Quotient Property of Square Roots.

$= \frac{\sqrt{12}\sqrt{x^6}}{\sqrt{25}\sqrt{y^2}}$ — 2 — Use the Product Property of Square Roots.

$= \frac{\sqrt{4}\sqrt{3}\sqrt{x^6}}{\sqrt{25}\sqrt{y^2}}$ — 3 — Write radicands as products with perfect squares.

$= \frac{2\sqrt{3}x^3}{5y}$ — 4 — Simplify.

Answer each question about the procedure shown above.

1. Which property is used to "separate" the fraction under a radical into two radicals?

Quotient Property of Square Roots

2. In step 3, which property is used to write $\sqrt{12}$ as $\sqrt{4}\sqrt{3}$?

Product Property of Square Roots

3. Describe the rule of exponents from Chapter 7 that is used to simplify $\sqrt{x^6}$ to x^3.

When multiplying powers with the same base, add the exponents.

So $x^3 \cdot x^3 = x^6$.

Simplify each radical expression.

4. $\sqrt{36x^2y^5}$ $6xy^2\sqrt{y}$

5. $\sqrt{\frac{8x^{10}}{36x^4}}$ $\frac{x^3\sqrt{2}}{3}$

6. $\sqrt{\frac{18x}{49y^8}}$ $\frac{3\sqrt{2x}}{7y^4}$

7. $\sqrt{44x^4y^4}$ $2\sqrt{11}x^2y^2$

8. $\sqrt{\frac{32x^5}{9y^8}}$ $\frac{4x^2\sqrt{2x}}{3y^4}$

9. $\sqrt{\frac{200x^{10}}{100}}$ $\sqrt{2}x^5$

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