LESSON 7-3

Practice A

Multiplication Properties of Exponents

Complete each equation to show the property.

1. $a^m \cdot a^n =$ a^{m+n}
2. $(a^m)^n =$ a^{mn}
3. $(ab)^n =$ $a^n b^n$

Simplify.

4. $7^5 \cdot 7^8$ — 7^{5+8}, 7^{13}
5. $3^{-2} \cdot 3^7$ — 3^{-2+7}, 3^5
6. $x^3 \cdot x^{-5}$ — $x^{3+(-5)}$, x^{-2}
7. $a^2 \cdot b^6 \cdot b^{-2}$ — $a^2 b^4$
8. $n^5 \cdot n^{-1} \cdot n^{-3}$ — n
9. $r^{-4} \cdot s^5 \cdot r^6$ — $r^2 s^5$
10. $(2^3)^4$ — $2^{3 \cdot 4}$, 2^{12}
11. $(4^0)^5$ — $4^{0 \cdot 5}$, 4^0, 1
12. $(b^2)^{-4}$ — $b^{2(-4)}$, b^{-8}, $\frac{1}{b^8}$
13. $3n)^4$ — $3^4 n^4$, $81n^4$
14. $(2x)^3$ — $2^3 x^3$, $8x^3$
15. $(t^3)^4 \cdot t^3$ — $t^{3 \cdot 4} \cdot t^3$, $t^{12} \cdot t^3$, t^{12+3}
16. $(b^4)^2 \cdot (b^3)^3 \cdot b^{-2}$ — b^{15}
17. $(a^2)^3 \cdot a^{-6}$ — 1
18. $(c^{-5})^2 \cdot (c^3)^{-2}$ — $\frac{1}{c^{16}}$
19. The volume of a cube can be found by using the formula $V = s^3$, where s represents the length of each side of the cube. Find the volume of a cube if each side is $4a^2b^4$.

$64a^6b^{12}$

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LESSON 7-3

Practice B

Multiplication Properties of Exponents

Simplify.

1. $3^4 \cdot 3^2$ — 3^6 or 729
2. $2^5 \cdot 2^4$ — 2^9 or 512
3. $2^3 \cdot 2^5 \cdot 2^1$ — 2^9 or 512
4. $q^{-6} \cdot q^{-1}$ — $\frac{1}{q^7}$
5. $r^{-3} \cdot r^4 \cdot s^{-4}$ — $\frac{r}{s^4}$
6. $j^{-2} \cdot j^{-4} \cdot j^2$ — $\frac{1}{j^4}$
7. $c^5 \cdot b^{-2} \cdot c^3$ — $\frac{c^8}{b^2}$
8. $(h^2)^5$ — h^{10}
9. $(g^4)^{-2}$ — $\frac{1}{g^8}$
10. $(w^6)^0$ — 1
11. $(v^2)^5 \cdot v^4$ — v^{14}
12. $(w^5)^{-2} \cdot w^{-3}$ — $\frac{1}{w^{13}}$
13. $(f^6)^{-4} \cdot (f^{-2})^{-3}$ — $\frac{1}{f^{18}}$
14. $(a^{-2})^{-3} \cdot (a^5)^2$ — a^{16}
15. $(3b)^4$ — $81b^4$
16. $(-5k)^2$ — $25k^2$
17. $-(4m)^3$ — $-64m^3$
18. $(-3p)^{-2}$ — $\frac{1}{9p^2}$
19. $(s^4 t)^3 \cdot (s^4 t^3)^2$ — $s^{20} t^9$
20. $(a^2 b^4)^2 \cdot (a^{-2} b^3)^{-1} \cdot a^4$ — $a^{10} b^5$
21. $(x^3 y^2)^{-4} \cdot (x^2 y^{-3})^{-2}$ — $\frac{1}{x^{16} y^2}$
22. The pitch of a sound is determined by the number of vibrations produced per second. The note "middle C" produces 2.62×10^2 vibrations per second. If a pianist plays middle C for 5×10^{-1} seconds, how many vibrations will occur?

1.31×10^2 or 131 vibrations

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LESSON 7-3

Practice C

Multiplication Properties of Exponents

Simplify.

1. $2^3 \cdot 2^5$ — 2^8 or 256
2. $6^4 \cdot 6^1$ — 6^5 or 7776
3. $5^6 \cdot 5^{-2}$ — 5^4 or 625
4. $t^{-4} \cdot t^{-5}$ — $\frac{1}{t^9}$
5. $x^4 \cdot x^2$ — x^6
6. $s^4 \cdot s^6 \cdot t^3$ — $s^{10} t^3$
7. $(2^4)^2$ — 2^8 or 256
8. $(m^4)^0$ — 1
9. $(y^{-2})^{-4}$ — y^8
10. $m^4 \cdot (n^2)^3 \cdot n^{-2}$ — $m^4 n^4$
11. $y \cdot x^3 \cdot (y^2)^2 \cdot (x^3)^6$ — $x^{21} y^5$
12. $(4x)^3$ — $64x^3$
13. $(gh)^{-1}$ — $\frac{1}{gh}$
14. $-(3bc)^2$ — $-9b^2 c^2$
15. $(j^2 k^3)^3$ — $j^6 k^9$
16. $(5d)^2 \cdot d^4$ — $25d^6$
17. $(3xy^3)^{-2} \cdot (9y)^2$ — $\frac{9}{x^2 y^4}$
18. $(r^{-3} s^4)^2 \cdot (r^2 s^7)^0$ — $\frac{s^8}{r^6}$
19. $(a^2 b^{-2})^{-3} \cdot (a^{-1} b^2)^{-3}$ — $\frac{1}{a^3}$
20. $(x^3 y^{-1})^4 \cdot (x^{-2} y^2)^{-3}$ — $\frac{x^{18}}{y^{10}}$
21. $-(j^{-3} k^{-2})^2 \cdot (j^2)^6$ — $-\frac{j^6}{k^4}$

Find the missing exponent in each expression.

22. $b^{\boxed{4}} \cdot b^5 = b^9$
23. $\left(t^{\boxed{3}}\right)^4 = t^{12}$
24. $(h^3)^{\boxed{-2}} = \frac{1}{h^6}$
25. Most states are irregularly shaped. However, the shapes of some western states approximate rectangles. Wyoming is nearly rectangular, with a width of about 1.5×10^6 feet and a length of about 1.9×10^6 feet. What is the approximate area of Wyoming? Write your answer in scientific notation.

2.85×10^{12} square feet

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LESSON 7-3

Reteach

Multiplication Properties of Exponents

You can multiply a power by a power by expanding each factor.

Simplify $(4^3)(4^5)$.

$(4^3)(4^5)$

$(4 \cdot 4 \cdot 4)(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)$ *Expand each factor.*

4^8 *Count the number of factors. The number of factors is the exponent.*

Or you can use the **Product of Powers Property**:

$a^m \cdot a^n = a^{m+n}$ ($a \neq 0$, m and n are integers.)

Simplify $(4^3)(4^5)$.

$(4^3)(4^5)$

4^{3+5}

4^8

Simplify $a^4 \cdot b^5 \cdot a^{-2}$.

$a^{4+(-2)} \cdot b^5$

$a^2 \cdot b^5$

$a^2 b^5$

You can use the **Power of a Power Property** to find a power raised to another power.

$(a^m)^n = a^{mn}$ ($a \neq 0$, m and n are integers.)

Simplify $(2^3)^2$.

$(2^3)^2$

$2^{3 \cdot 2}$

2^6

Simplify $(x^5)^4 \cdot y$.

$x^{5 \cdot 4} \cdot y$

$x^{20} y$

Simplify.

1. $2^3 \cdot 2^4$ — 2^7
2. $8^{-2} \cdot 5^3 \cdot 8^6$ — $8^4 \cdot 5^3$
3. $2^4 \cdot 3^5 \cdot 2^8 \cdot 3^{-2}$ — $2^{12} \cdot 3^3$
4. $m^8 \cdot n^4 \cdot m^7$ — $m^{15} n^4$
5. $(6^4)^2$ — 6^8
6. $(4^{-3})^2$ — $\frac{1}{4^6}$
7. $(5^{-3})^3 \cdot 4^0$ — $\frac{1}{5^9}$
8. $(x^2)^{-4} \cdot y^{-3}$ — $\frac{1}{x^8 y^3}$
9. $(u^5)^{-2} \cdot (v^3)^4$ — $\frac{v^{12}}{u^{10}}$

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LESSON 7-3

Reteach

Multiplication Properties of Exponents (continued)

In the **Power of a Product Property,** each factor is raised to that power.

$$(ab)^n = a^n b^n \quad (a \neq 0,\ b \neq 0,\ n \text{ is any integer.})$$

Simplify $(x^3y^{-5})^2$.

$(x^3y^{-5})^2$

$x^{3 \cdot 2} \cdot y^{-5 \cdot 2}$ *Use the Power of a Product Property.*

x^6y^{-10} *Simplify.*

$\frac{x^6}{y^{10}}$ *Write with positive exponents.*

Exponential expressions are simplified if:

- there are no negative exponents.
- the same base does not appear more than once in a product or a quotient.
- no powers, products or quotients are raised to powers.
- all fractions have been simplified.

Simplified				Not Simplified			
$\frac{x}{y}$	a^2b^3	$\frac{m^3}{n^3}$	$\frac{2g}{3h^4}$	x^{-2}	$(y^2)^4$	$(st)^4$	$\frac{2d^4}{6}$

Tell if each expression is simplified. If not, simplify.

10. $\frac{-3a^2}{8b}$ — yes

11. $(2h^3)^2$ — no; $4h^6$

12. $m^3 \cdot m^0$ — no; m^3

Simplify.

13. $(-4x^5)^2$ — $16x^{10}$

14. $(s^4t^3)^3$ — $s^{12}t^9$

15. $(-2x^{-4}y)^5$ — $\frac{-32y^5}{x^{20}}$

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LESSON 7-3

Challenge

Using Exponents to Understand Multiplication of Decimals

When you learned how to multiply one decimal by another, you learned to count decimal places and move the decimal point that many places to the left.

$0.003 \times 0.02 \longrightarrow$ Write 6 and move the decimal point 5 places to the left. $\longrightarrow 0.00006$

Using properties of exponents, you can understand why this rule works.

Find each product by counting decimal places and moving the decimal point.

1. 0.06×0.002 — 0.00012

2. 0.04×0.012 — 0.00048

3. 0.15×0.0006 — 0.00009

4. 0.09×0.00012 — 0.0000108

You can also find the product 0.003×0.02 by using a property of exponents that you learned. Notice that the final answer shown below agrees with the answer obtained by applying the rule for multiplication shown in the example above.

$$0.003 \times 0.02 = \frac{3}{1000} \times \frac{2}{100} = \frac{3}{10^3} \times \frac{2}{10^2} = \frac{3 \times 2}{10^3 \times 10^2} = \frac{3 \times 2}{10^{3+2}} = \frac{6}{10^5} = \frac{6}{10{,}000} = 0.00006$$

Find each product by using a property of exponents as shown above. Show your work.

5. 0.06×0.002 — 0.00012

6. 0.04×0.012 — 0.00048

7. 0.15×0.0006 — 0.00009

8. 0.09×0.00012 — 0.0000108

Find each product by using a property of exponents. Show your work.

9. $0.06 \times 0.002 \times 0.003$ — 0.00000036

10. $0.04 \times 0.05 \times 0.003$ — 0.000006

11. Write an extension of the rule for multiplying two decimals between 0 and 1 that applies to multiplying three such decimals.

Multiply the decimals in the same way as the whole numbers. Count the number of decimal places in each of the three numbers. Find the sum of those numbers. Move the decimal point that many places to the left.

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LESSON 7-3

Problem Solving

Multiplication Properties of Exponents

Write the correct answer.

1. In the mid-nineteenth century, several landowners in Australia released domestic rabbits into the wild. Suppose 100 rabbits were released. By 1950, the population had increased about 6×10^6 times. Determine the wild rabbit population in 1950.

about 600,000,000

2. Barnard's star is the fifth closest star to the Earth, after the Sun and the stars in the Alpha Centauri system. It takes 1.86×10^8 seconds for light from Barnard's star to reach the Earth. Light travels at a speed of 1.86×10^5 miles per second. Calculate the distance from Barnard's star to the Earth.

3.46×10^{13} miles

3. Saturn's smallest moon, Tethys, has a diameter of about 6.5×10^2 miles. The diameter of Jupiter's largest moon, Ganymede, is 5 times that of Tethys. Determine the diameter of Ganymede. Write your answer in standard form and in scientific notation.

about 3250 mi or 3.25×10^3 mi

4. Delaware and Montana have roughly the same population. Delaware's area is 2.49×10^3 square miles. Montana is 59 times larger. Determine the area of Montana. Write your answer in standard form and in scientific notation.

147,000 sq mi or 1.47×10^5 sq mi

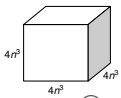
Select the best answer.

5. The formula for the volume of a cylinder is $V = 2\pi r^2 h$ where r is the radius and h is the height. What is the volume of the cylinder shown below?



A $12\pi xy$ cm^3 C $24\pi x^2y$ cm^3
B $12\pi xy^2$ cm^3 (D) $36\pi xy^2$ cm^3

6. What is the volume of the cube shown below?



F $12n^6$ in^3 (H) $64n^9$ in^3
G $12n^9$ in^3 J $256n^9$ in^3

7. Belize borders Mexico and Guatemala in Central America. It has an area of 2.30×10^4 square kilometers. Russia borders fourteen countries and is 7.43×10^2 times larger than Belize. What is the area of Russia?

A 1.71×10^6 sq km C 1.71×10^8 sq km
(B) 1.71×10^7 sq km D 1.71×10^9 sq km

8. In 1989, Voyager 2 discovered six moons that orbit Neptune. The smallest of these is Naiad, which orbits Neptune in a brief 7.2 hours, or 8.22×10^{-4} years. Neptune's orbit of the Sun takes 2×10^5 times longer than Naiad's. How long does Neptune's orbit take?

F 10.2 years H 102 years
G 16.4 years (J) 164 years

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LESSON 7-3

Reading Strategies

Use a Table

The table below summarizes the multiplication properties that are needed to simplify expressions with powers.

Product of a Power	To multiply powers with the same base, keep the base and **add the exponents.**	$4^3 \cdot 4^5 = 4^{3+5} = 4^8$ $3^8 \cdot 5^6 \cdot 3^{-6} = 3^{8+(-6)} \cdot 5^6 = 3^2 \cdot 5^6$ $x^{-5} \cdot x^4 \cdot y^7 = x^{-5+4} \cdot y^7 = x^{-1}y^7 = \frac{y^7}{x}$	In general, $a^m \cdot a^n = a^{m+n}$
Power of a Power	To find the power of a power, keep the base and **multiply the exponents.**	$(4^3)^5 = 4^{3 \cdot 5} = 4^{15}$ $(8^9)^0 = 8^{9 \cdot 0} = 8^0 = 1$ $(x^2)^{-6} = x^{2 \cdot -6} = x^{-12} = \frac{1}{x^{12}}$	In general, $(a^m)^n = a^{m \cdot n}$
Power of a Product	To find the power of a product, **apply the exponent** to each factor.	$(5 \cdot 3)^2 = 5^2 \cdot 3^2 = 25 \cdot 9 = 225$ $(-4b)^3 = (-4)^3 \cdot b^3 = -64b^3$ $(x^0 \cdot y^{-1})^5 = (x^0)^5 \cdot (y^{-1})^5 = x^0 \cdot y^{-5} = \frac{1}{y^5}$	In general, $(ab)^n = a^n b^n$

Complete each of the following.

1. What do you do with the exponents to simplify $(c^4)^{-2}$? — multiply

2. What do you do with the exponents to simplify $6^4 \cdot 6^8$? — add

3. What is the name of the property that can be used to simplify $(6t)^{-9}$? — Power of a Product

4. $3(x + 4) = 3x + 12$ shows how the Distributive Property of Multiplication is used to simplify an expression. Which property of exponents is similar to the Distributive Property? Why?

Power of a Product; with both properties, a number is applied to all parts.

Simplify each expression.

5. $(m^3)^8$ — m^{24}

6. $8^4 \cdot 8^4$ — 8^8

7. $(3v^5)^2$ — $9v^{10}$

8. $c^{-2} \cdot d^8 \cdot c^{-3}$ — $\frac{d^8}{c^5}$

9. $(2 \cdot 9)^3$ — 5832

10. $-(4y^7)^2$ — $-16y^{14}$

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