LESSON 7-4

Practice A

Division Properties of Exponents

Simplify.

1. $\frac{3^6}{3^2} = 3^{6-2} = 3^{\boxed{4}} =$ **81**

2. $\frac{b^8}{b^5} = b^{\boxed{8}-\boxed{5}} = b^{\boxed{3}}$

3. $\frac{t^2}{t^7} = t^{\boxed{2}-\boxed{7}} = t^{\boxed{-5}} =$ $\frac{1}{t^5}$

4. $\frac{s^3 t^4}{(s^2)^3} = \frac{s^3 t^4}{s^{\boxed{6}}} = s^{\boxed{3}-\boxed{6}} \cdot t^{\boxed{4}} =$ $\frac{t^4}{s^3}$

5. $\frac{(ab)^2}{(a^5 b^2)^3}$ $\frac{1}{a^{13} b^4}$

6. $\frac{x^3 y}{(x^2 y^3)^2}$ $\frac{1}{xy^5}$

7. $\frac{4^3 \cdot 3^4 \cdot 2^3}{3^2 \cdot 4^4 \cdot 2}$ **9**

8. $\left(\frac{2}{3}\right)^4 = \frac{2^{\boxed{4}}}{3^{\boxed{4}}} =$ $\frac{\boxed{16}}{\boxed{81}}$

9. $\left(\frac{x}{4}\right)^3 = \frac{x^{\boxed{3}}}{4^{\boxed{3}}} =$ $\frac{x^3}{64}$

10. $\left(\frac{4}{5}\right)^{-2} = \left(\frac{\boxed{5}}{\boxed{4}}\right)^{\boxed{2}} =$ $\frac{\boxed{25}}{\boxed{16}}$

11. $\left(\frac{3ab}{4c^2}\right)^4$ $\frac{81a^4 b^4}{256c^8}$

12. $\left(\frac{2b}{3c}\right)^{-3}$ $\frac{27c^3}{8b^3}$

13. $\left(\frac{3z^3}{4x^2 y}\right)^{-4} = \left(\frac{\boxed{4x^2 y}}{\boxed{3z^3}}\right)^{\boxed{4}} =$ $\frac{256x^8 y^4}{81z^{12}}$

14. $\left(\frac{n}{4}\right)^{-2} \cdot \left(\frac{6}{3n}\right)^{-3} = \left(\frac{\boxed{4}}{\boxed{n}}\right)^{\boxed{2}} \cdot \left(\frac{\boxed{3n}}{\boxed{6}}\right)^{\boxed{3}} =$ **2n**

Simplify. Write each answer in scientific notation.

15. $(4.5 \times 10^6) \div (3 \times 10^{-2})$

$\frac{4.5 \times 10^6}{3 \times 10^{-2}} = \frac{4.5}{3} \times \frac{10^6}{10^{-2}}$

$= \underline{1.5} \times 10^{\boxed{8}}$

16. $(1.2 \times 10^5) \div (6 \times 10^3)$

$\frac{1.2 \times 10^5}{6 \times 10^3} = \frac{1.2}{6} \times \frac{10^5}{10^3}$

$= \underline{0.2} \times 10^{\boxed{2}}$

$= (\underline{2} \times 10^{\boxed{-1}}) \times 10^{\boxed{2}}$

$= \underline{2 \times 10^1}$

17. A retired business man calculated that he earned $\$8.75 \times 10^6$ after working for 35 years.

   a. Write the number of years in scientific notation. 3.5×10^1

   b. Divide to find the average amount he earned per year. Write your answer in standard form. \$250,000

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LESSON 7-4

Practice B

Division Properties of Exponents

Simplify.

1. $\frac{6^7}{6^5} = 6^{7-5} = 6^{\boxed{2}} =$ **36**

2. $\frac{t^{12}}{t^7} = t^{\boxed{12}-\boxed{7}} =$ t^5

3. $\frac{w^9}{w^2}$ w^7

4. $\frac{j^2}{j^8}$ $\frac{1}{j^6}$

5. $\frac{20m^5}{4m^2}$ $5m^3$

6. $\frac{c^3 d^2}{c^2 d^5}$ $\frac{c}{d^3}$

7. $\frac{(x^4)^2}{(x^3)^5}$ $\frac{1}{x^7}$

8. $\left(\frac{s^3 t}{st^4}\right)^2$ $\frac{s^4}{t^6}$

9. $\left(\frac{2}{3}\right)^{-3}$ $\frac{27}{8}$

10. $\left(\frac{3a}{2b}\right)^{-4}$ $\frac{16b^4}{81a^4}$

11. $-\left(\frac{-t}{3v}\right)^{-4}$ $\frac{-81v^4}{t^4}$

12. $\left(\frac{6}{7}\right)^{-2} \cdot \left(\frac{4s}{6t}\right)^{-2}$ $\frac{49t^2}{16s^2}$

13. $\left(\frac{3c}{-2}\right)^{-1} \left(\frac{d}{4}\right)^{-2}$ $\frac{-32}{3cd^2}$

14. $\left(\left(\frac{3mn}{2}\right)^{-1}\right)^{-4}$ $\frac{81m^4 n^4}{16}$

Simplify. Write the answer in scientific notation.

15. $(3.8 \times 10^5) \div (1.9 \times 10^{-6})$ 2×10^{11}

16. $(2.5 \times 10^3) \div (5 \times 10^{-4})$ 5×10^6

17. A textile factory produces 1.08×10^8 yards of fabric every year. If the factory is in operation 360 days a year, what is the average number of yards of fabric produced each day? Give your answer in standard form. **300,000 yards**

18. It takes 5 yards of fabric to manufacture a dress. If the textile factory turned their entire yearly production of 1.08×10^8 yards of fabric into dresses, how many could they make? Give your answer in scientific notation. 2.16×10^7 **dresses**

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LESSON 7-4

Practice C

Division Properties of Exponents

Simplify.

1. $\frac{6^8}{6^6}$ 6^2 **or** 36

2. $\frac{h^4}{h^{-3}}$ h^7

3. $\frac{2^3 \cdot 4^3 \cdot 5}{5^2 \cdot 2^4}$ $\frac{32}{5}$

4. $\frac{x^5 y^2}{xy^3}$ $\frac{x^4}{y}$

5. $\frac{m^3 n^6}{m^4 n^4 p^8}$ $\frac{n^2}{mp^8}$

6. $\frac{a^5 b^2 c^3}{a^6 b^2 c}$ $\frac{c^2}{a}$

7. $\left(\frac{4}{7}\right)^{-2}$ $\frac{49}{16}$

8. $\left(\frac{s^2}{t^3}\right)^2 =$ $\frac{s^4}{t^6}$

9. $-\left(\frac{ab}{6c}\right)^5$ $\frac{-a^5 b^5}{7776c^5}$

10. $-\left(\frac{b^2 c}{2d^3 f^4}\right)^{-2}$ $\frac{-4d^6 f^8}{b^4 c^2}$

11. $\left(\frac{xyz^2}{-w}\right)^5 =$ $\frac{-x^5 y^5 z^{10}}{w^5}$

12. $\left(\frac{10^3 \cdot 10^2}{10^{-6}}\right)^{-4}$ $\frac{1}{10^{44}}$

Simplify. Write the answer in scientific notation.

13. $(6.4 \times 10^7) \div (1.6 \times 10^3)$ 4×10^4

14. $(8.1 \times 10^{-6}) \div (9 \times 10^{-15})$ 9×10^8

15. $(2.8 \times 10^2) \div (7 \times 10^{-9})$ 4×10^{10}

16. $(4.8 \times 10^5) \div (6 \times 10^{13})$ 8×10^{-9}

Find the missing exponent in each equation.

17. $\left(c^{\boxed{4}}\right)^{-2} = \frac{d^2}{c^8}$

18. $\frac{b^{14}}{b^{\boxed{7}}} = b^7$

19. $\left(\frac{s^{-3}}{t^4}\right)^{\boxed{-3}} = s^9 t^{12}$

20. An actor was paid $\$2.1 \times 10^6$ to star in a movie. If the movie was 1 hr 45 min long, what was the actor's salary per minute? Give your answer standard form. **\$20,000 per minute**

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LESSON 7-4

Reteach

Division Properties of Exponents

The **Quotient of Powers Property** can be used to divide terms with exponents.

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0,\ m \text{ and } n \text{ are integers.})$$

Simplify $\frac{7^5}{7^2}$.

$\frac{7^5}{7^2} = 7^{5-2}$

$= 7^3$

Simplify $\frac{x^7 y}{x^3}$.

$\frac{x^7 y}{x^3} = x^{7-3} \cdot y$

$= x^4 y$

The **Positive Power of a Quotient Property** can be used to raise quotients to positive powers.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (a \neq 0,\ b \neq 0,\ n \text{ is a positive integer.})$$

Simplify $\left(\frac{2}{5}\right)^4$.

$\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4}$ *Use the Positive Power of a Quotient Property.*

$= \frac{16}{625}$ *Simplify.*

Simplify $\left(\frac{2x^5}{y^4}\right)^3$.

$\left(\frac{2x^5}{y^4}\right)^3 = \frac{(2x^5)^3}{(y^4)^3}$ *Use the Positive Power of a Quotient Property.*

$= \frac{2^3 (x^5)^3}{(y^4)^3}$ *Use the Power of a Product Property.*

$= \frac{8x^{15}}{y^{12}}$ *Simplify.*

Simplify.

1. $\frac{5^6}{5^4}$ **25**

2. $\frac{x^6 y^5}{y^3}$ $x^6 y^2$

3. $\frac{a^2 b^4}{(ab)^3}$ $\frac{b}{a}$

4. $\left(\frac{2}{5}\right)^3$ $\frac{2^3}{5^3}$ **or** $\frac{8}{125}$

5. $\left(\frac{x^3}{y^2}\right)^6$ $\frac{x^{18}}{y^{12}}$

6. $\left(\frac{3m^3}{n^2}\right)^2$ $\frac{9m^6}{n^4}$

7. $\left(\frac{a}{b^2}\right)^3$ $\frac{a^3}{b^6}$

8. $\left(\frac{x^3}{xy}\right)^2$ $\frac{x^4}{y^2}$

9. $\left(\frac{30}{20}\right)^2$ $\frac{9}{4}$

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LESSON 7-4

Reteach

Division Properties of Exponents (continued)

You can divide quotients raised to a negative power by using the **Negative Power of a Quotient Property**.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n} \quad (a \neq 0,\ b \neq 0,\ n \text{ is a positive integer})$$

Simplify $\left(\frac{3}{4}\right)^{-2}$.

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2}$	*Rewrite with a positive exponent.*
$= \frac{4^2}{3^2}$	*Use the Positive Power of a Quotient Property.*
$= \frac{16}{9}$	*Simplify.*

Simplify $\left(\frac{3a^4}{b^2}\right)^{-3}$.

$\left(\frac{3a^4}{b^2}\right)^{-3} = \left(\frac{b^2}{3a^4}\right)^{3}$	*Rewrite with a positive exponent.*
$= \frac{(b^2)^3}{(3a^4)^3}$	*Use the positive Power of a Quotient Property.*
$= \frac{b^{2 \cdot 3}}{3^3 a^{4 \cdot 3}}$	*Use the Power of a Power Property.*
$= \frac{b^6}{27a^{12}}$	*Simplify.*

Fill in the blanks below.

10. $\left(\frac{3}{5}\right)^{-3} = \left(\frac{\boxed{5}}{\boxed{3}}\right)^{3} = \frac{\boxed{5}^3}{\boxed{3}^3} = \frac{\boxed{125}}{\boxed{27}}$

11. $\left(\frac{xy^3}{z^7}\right)^{-5} = \left(\frac{\boxed{z^7}}{\boxed{xy^3}}\right)^{5} = \frac{z^{\boxed{7}\cdot 5}}{x^{\boxed{1}\cdot 5} y^{\boxed{3}\cdot 5}} = \frac{z^{\boxed{35}}}{x^{\boxed{5}} y^{\boxed{15}}}$

12. $\left(\frac{a^2b^3}{c}\right)^{-4} = \left(\frac{\boxed{c}}{\boxed{a^2b^3}}\right)^{4} = \frac{c^{\boxed{1}\cdot 4}}{a^{\boxed{2}\cdot 4} b^{\boxed{3}\cdot 4}} = \frac{c^{\boxed{4}}}{a^{\boxed{8}} b^{\boxed{12}}}$

Simplify.

13. $\left(\frac{x}{y}\right)^{-5}$ — $\frac{y^5}{x^5}$

14. $\left(\frac{7}{3m^4}\right)^{-2}$ — $\frac{9m^8}{49}$

15. $\left(\frac{2a^2}{b^3}\right)^{-5}$ — $\frac{b^{15}}{32a^{10}}$

16. $\left(\frac{m}{3n}\right)^{-2}$ — $\frac{9n^2}{m^2}$

17. $\left(\frac{2}{3x^2}\right)^{-3}$ — $\frac{27x^6}{8}$

18. $\left(\frac{r}{2s^3}\right)^{-4}$ — $\frac{16s^{12}}{r^4}$

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LESSON 7-4

Challenge

Applying Properties of Exponents to Rational Numbers

You can use the following three facts to discover a new and interesting fact about rational numbers:

- A rational number is the quotient of two integers with a nonzero denominator.
- Every integer can be written as a product of powers of prime numbers, called the *prime factorization* of the given number. For example, $120 = 2^3 3^1 5^1$.
- When dividing two powers with the same base, subtract the exponents. $\frac{10^5}{10^3} = 10^2$ and $\frac{10^3}{10^5} = \frac{1}{10^2}$

Write the prime factorization of each integer.

1. 24 — $2^3 \times 3^1$
2. 108 — $2^2 \times 3^3$
3. 452 — $2^2 \times 113^1$
4. 1800 — $2^3 \times 3^2 \times 5^2$

For each rational number, write the numerator and denominator by using the prime factorization of each. Then use the Quotient-of-Powers Property to simplify the result. Do not multiply out the powers of prime numbers that remain.

5. $\frac{18}{24}$ — $\frac{2 \cdot 3^2}{2^3 \cdot 3} = \frac{3}{2^2}$
6. $\frac{48}{180}$ — $\frac{2^4 \cdot 3}{2^2 \cdot 3^2 \cdot 5} = \frac{2^2}{3 \cdot 5}$
7. $\frac{250}{288}$ — $\frac{2 \cdot 5^3}{2^5 \cdot 3^2} = \frac{5^3}{2^4 \cdot 3^2}$
8. $\frac{540}{1800}$ — $\frac{2^2 \cdot 3^3 \cdot 5}{2^3 \cdot 3^2 \cdot 5^2} = \frac{3}{2 \cdot 5}$

9. Examine the final quotients that you wrote in Exercises 5–8. Explain why a prime-number base that appears in a numerator does not appear in the denominator and why a prime-number base that appears in a denominator does not appear in the numerator.

If a prime number base *b* appear in the numerator (or denominator), it cannot occur in the denominator (or numerator) as well because then the rational number is not fully simplified.

ex: $\frac{b^n a}{b^m c} = \left(\frac{b^m}{b^m}\right)\frac{b^{n-m}a}{c} = \frac{b^{n-m}a}{c}$

10. Let $\frac{a}{b}$ be a rational number. Write a generalization about the representation of $\frac{a}{b}$ as the quotient of prime numbers raised to powers. Illustrate your generalization by using $\frac{54}{120} = \frac{2^1 3^3}{2^3 3^1 5^1}$.

Every rational number can be written as a quotient whose numerator is 1 or the product of prime numbers raised to positive integer exponents and whose denominator can be written as 1 or the product of prime numbers raised to positive integer exponents, and there are no prime bases common to the numerator and the denominator.

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LESSON 7-4

Problem Solving

Division Properties of Exponents

Write the correct answer.

1. Kudzu is a fast-growing vine that has become a nuisance in the southeastern United States. It covers 2.5×10^5 acres in Alabama. In 2004 the population of Alabama was estimated to be 4.45×10^6 people. How many acres of kudzu are there for each person in Alabama?

0.056 acres

2. A cylindrical water tank has a volume of $6\pi x^2 y^4$ cubic meters. The formula for the volume of a cylinder is $\pi r^2 h$. The water tank has a radius of *xy* meters. What is its height?

$6y^2$ meters

3. Voyager 2 was launched in 1979 to explore the planets of the outer solar system. The spacecraft travels an average of 4.68×10^8 kilometers in one year. Determine the speed of Voyager 2 in kilometers per hour. (*Hint*: 1 year = 8760 hours)

5.34×10^4 km/hr

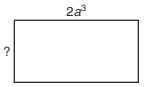
4. The population of Laos is 6.22×10^6. In 2004 its gross domestic product (GDP) was $\$1.13 \times 10^{10}$. The population of Norway is 4.59×10^6. In 2004 its GDP was $\$1.83 \times 10^{11}$. What is the GDP per capita, or per person, of Laos and Norway?

Laos: \$1817

Norway: \$39,869

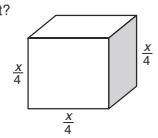
Select the best answer.

5. A rectangular parking lot has an area of $10a^3b^6$ square yards. What is the width of the parking lot?



A $5b^2$ yards (C) $5b^6$ yards
B $5b^3$ yards D $25b^6$ yards

6. A storage chest is shaped like a cube. What is the volume of the storage chest?



(F) $\frac{x^3}{64}$ cubic units H $\frac{32}{x^3}$ cubic units
G $\frac{x^3}{32}$ cubic units J $64x^3$ cubic units

7. The wavelengths of electromagnetic radiation vary greatly. Green light has a wavelength of about 5.1×10^{-7} meters. The wavelength of a U-band radio wave is 2.0×10^{-2} meters. About how much greater is the wavelength of a U-band radio wave than that of green light?

A 2.55×10^{-9} (C) 3.92×10^4
B 2.55×10^{-5} D 3.92×10^5

8. Puerto Rico has an area of 5.32×10^3 square miles and a population of 3.89×10^6. What is the population density of Puerto Rico in persons per square mile?

F 1.37×10^{-3} (H) 7.31×10^2
G 1.37×10^{-2} J 7.31×10^3

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LESSON 7-4

Reading Strategies

Use a Table

The table below summarizes the division properties that are needed to simplify expressions with powers.

Quotient of Powers	To divide powers with the same base, keep the base and **subtract the exponents.**	$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 16$ $\frac{x^{12}y^4}{xy^6} = x^{12-1} \cdot y^{4-6} = x^{11}y^{-2} = \frac{x^{11}}{y^2}$	In general, $\frac{a^m}{a^n} = a^{m-n}$
Positive Power of a Quotient	To find the positive power of a quotient, **apply the exponent** to the numerator and denominator.	$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$ $\left(\frac{x^2y^5}{y}\right)^3 = \frac{(x^2y^5)^3}{(y)^3} = \frac{x^6y^{15}}{y^3} = x^6y^{12}$	In general, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Negative Power of a Quotient	To find the negative power of a quotient, **apply the positive exponent** to the numerator and denominator of the **reciprocal**.	$\left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$ $\left(\frac{x^3y}{x^5}\right)^{-4} = \left(\frac{x^5}{x^3y}\right)^4 = \frac{x^{20}}{x^{12}y^4} = \frac{x^8}{y^4}$	In general, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

Complete each of the following.

1. What do you do with the exponents to simplify $\frac{b^{13}}{b^8}$? — **subtract**
2. Rewrite the expression $\left(\frac{5}{8}\right)^{-4}$ using a positive exponent. — $\left(\frac{8}{5}\right)^4$
3. What is the name of the property that can be used to simplify the expression $\left(\frac{4t}{9}\right)^3$? — **Positive Power of a Quotient**

Simplify each expression.

4. $\frac{12^{10}}{12^8}$ — 144
5. $\left(\frac{2}{5}\right)^4$ — $\frac{16}{625}$
6. $\left(\frac{9}{8}\right)^{-2}$ — $\frac{64}{81}$
7. $\frac{f^3g^4}{f^7gh^5}$ — $\frac{g^3}{f^4h^5}$
8. $\left(\frac{st}{t^4}\right)^{-6}$ — $\frac{t^{18}}{s^6}$
9. $\left(\frac{2c^2}{c^4d}\right)^5$ — $\frac{32}{c^{10}d^5}$
10. $\left(\frac{3}{2}\right)^{-3}$ — $\frac{8}{27}$
11. $\left(\frac{x^3y}{xy^3}\right)^2$ — $\frac{x^4}{y^4}$
12. $\left(\frac{5f^3}{g^7}\right)^{-2}$ — $\frac{g^{14}}{25f^6}$

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