LESSON 8-1

Practice A

Factors and Greatest Common Factors

Complete the prime factorization of each number.

1. 36 → 3 · 12 → 3 · 4 → 2 · 2

$3^2 \cdot 2^2$

2. 2|80, 2|40, 2|20, 2|10, 5

$2^4 \cdot 5$

3. 125 → 5 · 25 → 5 · 5

5^3

Find the GCF of each pair of numbers.

4. 15 and 40: 5

5. 8 and 32: 8

6. 36 and 48: 12

7. 50 and 75: 25

Find the GCF of each pair of monomials.

8. $12y^3$ and $15y^2$: $3y^2$

9. $3p^4$ and $4p$: p

10. $18x^6$ and $24y^2$: 6

11. $14xy^2$ and $21y^3$: $7y^2$

Mrs. Graham is creating student envelopes for a math activity in her class. She has 64 problems written on pieces of blue paper and 48 problems written on pieces of red paper. She needs to sort the pieces of paper so that each envelope has the same number of pieces and no envelope has both red and blue pieces.

12. If Mrs. Graham puts the greatest possible number of papers in each envelope, how many papers will go in each envelope? 16

13. How many envelopes can Mrs. Graham create if she puts the greatest possible number of papers in each envelope? 7

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LESSON 8-1

Practice B

Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 18: $2 \cdot 3^2$

2. 120: $2^3 \cdot 3 \cdot 5$

3. 56: $2^3 \cdot 7$

4. 390: $2 \cdot 3 \cdot 5 \cdot 13$

5. 144: $2^4 \cdot 3^2$

6. 153: $3^2 \cdot 17$

Find the GCF of each pair of numbers.

7. 16 and 20: 4

8. 9 and 36: 9

9. 15 and 28: 1

10. 35 and 42: 7

11. 33 and 66: 33

12. 100 and 120: 20

13. 78 and 30: 6

14. 84 and 42: 42

Find the GCF of each pair of monomials.

15. $15x^4$ and $35x^2$: $5x^2$

16. $12p^2$ and $30q^5$: 6

17. $-6t^3$ and $9t$: $3t$

18. $27y^3z$ and $45x^2y$: $9y$

19. $12ab$ and 12: 12

20. $-8d^3$ and $14d^4$: $2d^3$

21. $-m^8n^4$ and $3m^6n$: m^6n

22. $10gh^2$ and $5h$: $5h$

23. Kirstin is decorating her bedroom wall with photographs. She has 36 photographs of family and 28 photographs of friends. She wants to arrange the photographs in rows so that each row has the same number of photographs, and photographs of family and photographs of friends do not appear in the same row.

a. How many rows will there be if Kirstin puts the greatest possible number of photographs in each row? 16

b. How many photographs will be in each row? 4

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LESSON 8-1

Practice C

Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 75: $3 \cdot 5^2$

2. 160: $2^5 \cdot 5$

3. 3500: $2^2 \cdot 5^3 \cdot 7$

Find the GCF of each set of numbers.

4. 18 and 36: 18

5. 54 and 60: 6

6. 30 and 49: 1

7. 72 and 54: 18

8. 12, 18, and 30: 6

9. 8, 20, and 28: 4

10. 15, 20, and 42: 1

11. 16, 24 and 56: 8

Find the GCF of each set of monomials.

12. $21m^3$ and $28m$: $7m$

13. $13x$ and 26: 13

14. $8x^2y$ and $-12x^3y^2$: $4x^2y$

15. $30s^4t^4$ and $36s^3t^5$: $6s^3t^4$

16. $18t^5$, $120s^4$, and $30t^2$: 6

17. $4x^2$, $2x^6$, and $-x^3$: x^2

18. $-6m^5$, $5n^6$, and $8n$: 1

19. $35y^4z$, $-14y^3z^2$, and $-7y^2$: $7y^2$

20. Emilio has 30 oranges, 45 apples, and 20 pears with which to make fruit baskets. He wants each fruit basket to contain the same number of each type of fruit. If he puts the greatest possible number of each type of fruit into each basket, how many fruit baskets can he make? How many pieces of each type of fruit will be in each basket?

5 baskets; each will have 6 oranges, 9 apples, and 4 pears.

21. Nina is babysitting a little boy who has a collection of small vehicles. He has 36 cars, 12 vans, and 27 trucks. The boy is trying to line up the vehicles in rows, with the same number of vehicles in each row. He does not want different types of vehicles to be in the same row. Describe the arrangement of the rows if Nina helps the boy put the greatest possible number of vehicles in each row.

There will be 25 rows with 3 vehicles in each row.

There will be 12 rows of cars, 4 rows of vans, and 9 rows of trucks.

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LESSON 8-1

Reteach

Factors and Greatest Common Factors

A prime number has exactly two factors, itself and 1. The number 1 is not a prime number. To write the **prime factorization** of a number, factor the number into its prime factors only.

Find the prime factorization of 30.

Choose any prime number that is a factor of 30. Then divide.

5|30 → 5|30, 6 → 5|30, 2|6 → 5|30, 2|6, 3 → 5|30, 2|6, 3|3 → 5|30, 2|6, 3|3, 1

$30 = 5 \cdot 2 \cdot 3$

Repeat the process with the quotient.

The prime factorization of 30 is $2 \cdot 3 \cdot 5$.

Find the prime factorization of 84.

2|84, 2|42, 3|21, 7|7, 1

Check by multiplying: $2 \cdot 2 \cdot 3 \cdot 7 = 84$

The prime factorization of 84 is $2 \cdot 2 \cdot 3 \cdot 7$ or $2^2 \cdot 3 \cdot 7$.

Fill in the blanks below to find the prime factorization of the given numbers.

1. 2|44, 2|22, 11|11, 1: $2^2 \cdot 11$

2. 2|56, 2|28, 2|14, 7|7, 1: $2^3 \cdot 7$

3. 3|81, 3|27, 3|9, 3|3, 1: 3^4

Write the prime factorization of each number.

4. 99: $3^2 \cdot 11$

5. 75: $3 \cdot 5^2$

6. 84: $2^2 \cdot 3 \cdot 7$

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LESSON 8-1

Reteach

Factors and Greatest Common Factors (continued)

If two numbers have the same factors, the numbers have common factors.

The largest of the common factors is called the **greatest common factor**, or GCF.

Find the GCF of 12 and 18.

Think of the numbers you multiply to equal 12.

$1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$

The factors of 12 are: **1**, **2**, **3**, 4, **6**, 12

Think of the numbers you multiply to equal 18.

$1 \times 18 = 18$
$2 \times 9 = 18$
$3 \times 6 = 18$

The factors of 18 are: **1**, **2**, **3**, **6**, 9, 18.

The GCF of 12 and 18 is 6.

Find the GCF of $8x^2$ and $10x$.

The factors of $8x^2$ are: **1**, **2**, 4, 8, ***x***, *x*

The factors of $10x$ are: **1**, **2**, 5, 10, ***x***

2 ***x***

The GCF of $8x^2$ and $10x$ is $2x$.

Find the GCF of 28 and 44 by following the steps below.

7. Find the factors of 28. 1, 2, 4, 7, 14, 28

8. Find the factors of 44. 1, 2, 4, 11, 22, 44

9. Find the GCF of 28 and 44. 4

Find the GCF of each pair of numbers.

10. 15 and 20: 5

11. 16 and 28: 4

12. 24 and 60: 12

Find the GCF of each pair of monomials.

13. $4a$ and $10a$: $2a$

14. $15x^3$ and $21x^2$: $3x^2$

15. $5y^2$ and $8y$: y

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LESSON 8-1

Challenge

Using Prime Factorization to Find the GCF

There are many reasons why you might need to factor a number. Small numbers are easily factorable, and you can write their factors with a small amount of work. For large numbers, such as 180, you can use the method shown at right. Notice that the method is based on successive division by small prime numbers. First try 2 as a divisor, then try 3, and so on. From the division shown at right, you can write the prime factorization of 180 as follows.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 3^2 5$$

2	180
2	90
3	45
3	15
5	5
	1

Using the method shown above, find the prime factorization of each number.

1. 825: $3^1 5^2 11^1$

2. 1575: $3^2 5^2 7^1$

3. 1024: 2^{10}

4. 25,200: $2^4 3^2 5^2 7^1$

Once you have written two large numbers by using their prime factorization, you can easily find their common factors.

List the distinct prime-number factors of each pair of numbers. If the numbers have no common factors other than 1, write *relatively prime*.

5. $2^2 3^2 5^2 7^2$ and $2^1 3^3 5^9 7^4$: 2, 3, 5, and 7

6. $2^5 3^2 11^2 13^5$ and $2^1 11^3 19^9$: 2 and 11

7. $2^{10} 5^8 23^2 53^2$ and $3^1 7^3 17^9 19^4$: relatively prime

8. $5^5 3^2 17^2 31^5$ and $2^1 11^3 5^5 3^2 17^2 31^5$: 3, 5, 17, and 31

Once you have identified which prime numbers are common factors of two numbers, then find the highest powers of those prime factors that are common factors. The product of such powers gives the greatest common factor (GCF) of the two numbers.

Find the greatest common factor of each pair of numbers.

9. the pair in Exercise 5: $2^1 3^2 5^2 7^2$

10. the pair in Exercise 6: $2^1 11^2$

11. the pair in Exercise 7: 1

12. the pair in Exercise 8: $3^2 5^5 17^2 31^5$

13. 3,293,136 and 3,778,488: $2^3 3^4 7^1$

14. 2,662,000 and 26,159,679: 1

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LESSON 8-1

Problem Solving

Factors and Greatest Common Factors

Write the correct answer.

1. Eloise saved all her awards from school. She has 18 athletic awards and 27 academic awards. Eloise wants to display the two types separately but in rows of equal length. Determine the greatest number of awards Eloise can put in each row. Then determine the total number of rows.

9 awards in each row; total of 5 rows

2. Parker is preparing snacks for the children at a day camp. He has 48 carrot sticks and 36 apple slices. Find the number of identical snacks he can prepare if he puts as much food into each snack as possible without any leftovers. Then describe the snack.

12 snacks of 4 carrot sticks and 3 apple slices

3. Matías and Hannah are responsible for the centerpieces on the buffet tables at the school dance. They have 6 dozen carnations, 80 lilies, and 64 rosebuds. All the centerpieces must be identical. Determine the greatest number of centerpieces Matías and Hannah can make if they use all the flowers. Then describe the centerpiece.

8 centerpieces; 9 carnations, 10 lilies, 8 rosebuds

4. Ms. Thompson has 120 same-sized tiles with which to make a rectangular design on her patio floor. 54 of the tiles are green; the rest are blue. To keep the rectangular shape, all rows must have the same number of tiles. If she wants each row to have the same number of blue and green tiles, how many rows will there be if she wants to use all the tiles and wants the rows to be as long as possible?

6 rows

Part of an ad for interlocking foam squares is shown below. Use it to answer questions 5–7. Select the best answer.

5. A class arranges one package into a rectangle with dimensions other than those shown. Which could have been the dimensions?

(A) 2×18 **C** 4×8
B 3×16 **D** 5×6

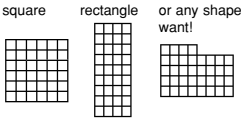
6. A teacher has 2 packages of red, 6 packages of blue, and half a package of yellow squares. He wants to build a rectangle so that each row is the same color. What can be the maximum number of squares per row?

F 2 (H) 18
G 6 **J** 36

Each package comes with 36 foam squares that interlock for a safe, colorful floor mat!

You can make a ...

square rectangle or any shape you want!



7. In problem 6, how many rows will be blue?

A 2 **C** 6
B 3 (D) 12

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LESSON 8-1

Reading Strategies

Focus on Vocabulary

There are many vocabulary words related to the concepts of GCF and factoring.

Factor: a number that divides evenly into another number
Ex: All the factors of 12 are 1, 2, 3, 4, 6, and 12.

Prime Number: a number that has exactly two factors, 1 and itself
Ex: The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Factorization: the factors of a number written as a product
Ex: $15 \rightarrow 1 \cdot 15$ and $3 \cdot 5$

Prime Factorization: the factors of a number written as a product of prime numbers
Ex: $18 \rightarrow 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$

Greatest Common Factor (GCF): the largest factor of two or more numbers
Ex: The GCF of 12 and 18 is 6. The GCF of $14x^3$ and $8x^2$ is $2x^2$.

Answer the following.

1. List the factors of 28. 1, 2, 4, 7, 14, 28

2. Write the factorization of a^5. $a \cdot a \cdot a \cdot a \cdot a$

3. List the factors of 16. 1, 2, 4, 8, 16

4. Write the factorization of a^2. $a \cdot a$

5. What is the GCF of the monomials $28a^5$ and $16a^2$? $4a^2$

6. What is the GCF of the monomials $34m^3$ and $8m$? $2m$

7. Is 33 a prime number? Explain. no; 11 and 3 are factors.

Find the prime factorization of each number.

8. 54: $2 \cdot 3^3$

9. 150: $2 \cdot 3 \cdot 5^2$

10. 80: $2^4 \cdot 5$

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