LESSON 8-3

Practice A

Factoring $x^2 + bx + c$

Factor each trinomial.

1. $x^2 + 5x + 6$ — $(x + 3)(x + 2)$
2. $x^2 + 5x + 4$ — $(x + 4)(x + 1)$
3. $x^2 + 9x + 20$ — $(x + 5)(x + 4)$
4. $x^2 + 10x + 21$ — $(x + 7)(x + 3)$
5. $x^2 + 11x + 30$ — $(x + 6)(x + 5)$
6. $x^2 + 10x + 16$ — $(x + 8)(x + 2)$
7. $x^2 - 8x + 12$ — $(x - 6)(x - 2)$
8. $x^2 - 8x + 15$ — $(x - 5)(x - 3)$
9. $x^2 - 17x + 16$ — $(x - 16)(x - 1)$
10. $x^2 - 12x + 27$ — $(x - 9)(x - 3)$
11. $x^2 - 15x + 44$ — $(x - 4)(x - 11)$
12. $x^2 - 13x + 40$ — $(x - 8)(x - 5)$
13. $x^2 + 6x - 40$ — $(x + 10)(x - 4)$
14. $x^2 + 2x - 3$ — $(x + 3)(x - 1)$
15. $x^2 + 4x - 32$ — $(x + 8)(x - 4)$
16. $x^2 + 10x - 24$ — $(x + 12)(x - 2)$
17. $x^2 + 12x - 28$ — $(x + 14)(x - 2)$
18. $x^2 + 3x - 10$ — $(x + 5)(x - 2)$
19. $x^2 - 2x - 15$ — $(x + 3)(x - 5)$
20. $x^2 - 8x - 20$ — $(x + 2)(x - 10)$
21. $x^2 - 2x - 48$ — $(x + 6)(x - 8)$
22. $x^2 - x - 12$ — $(x + 3)(x - 4)$
23. $x^2 - 2x - 3$ — $(x + 1)(x - 3)$
24. $x^2 - x - 2$ — $(x + 1)(x - 2)$

25. Factor $n^2 + 6n + 5$. Complete the tables to show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3$, and 4.

$(n + 1)(n + 5)$

n	$n^2 + 6n + 5$
0	$0^2 + 6(0) + 5 = 5$
1	$1^2 + 6(1) + 5 = 12$
2	$2^2 + 6(2) + 5 = 21$
3	$3^2 + 6(3) + 5 = 32$
4	$4^2 + 6(4) + 5 = 45$

n	$(n + 1)(n + 5)$
0	$(0 + 1)(0 + 5) = 5$
1	$(1 + 1)(1 + 5) = 12$
2	$(2 + 1)(2 + 5) = 21$
3	$(3 + 1)(3 + 5) = 32$
4	$(4 + 1)(4 + 5) = 45$

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LESSON 8-3

Practice B

Factoring $x^2 + bx + c$

Factor each trinomial.

1. $x^2 + 7x + 10$ — $(x + 2)(x + 5)$
2. $x^2 + 9x + 8$ — $(x + 1)(x + 8)$
3. $x^2 + 13x + 36$ — $(x + 4)(x + 9)$
4. $x^2 + 9x + 14$ — $(x + 7)(x + 2)$
5. $x^2 + 7x + 12$ — $(x + 3)(x + 4)$
6. $x^2 + 9x + 18$ — $(x + 6)(x + 3)$
7. $x^2 - 9x + 18$ — $(x - 6)(x - 3)$
8. $x^2 - 5x + 4$ — $(x - 4)(x - 1)$
9. $x^2 - 9x + 20$ — $(x - 5)(x - 4)$
10. $x^2 - 12x + 20$ — $(x - 2)(x - 10)$
11. $x^2 - 11x + 18$ — $(x - 9)(x - 2)$
12. $x^2 - 12x + 32$ — $(x - 8)(x - 4)$
13. $x^2 + 7x - 18$ — $(x + 9)(x - 2)$
14. $x^2 + 10x - 24$ — $(x + 12)(x - 2)$
15. $x^2 + 2x - 3$ — $(x + 3)(x - 1)$
16. $x^2 + 2x - 15$ — $(x + 5)(x - 3)$
17. $x^2 + 5x - 6$ — $(x + 6)(x - 1)$
18. $x^2 + 5x - 24$ — $(x + 8)(x - 3)$
19. $x^2 - 5x - 6$ — $(x + 1)(x - 6)$
20. $x^2 - 2x - 35$ — $(x + 5)(x - 7)$
21. $x^2 - 7x - 30$ — $(x + 3)(x - 10)$
22. $x^2 - x - 56$ — $(x + 7)(x - 8)$
23. $x^2 - 2x - 8$ — $(x + 2)(x - 4)$
24. $x^2 - x - 20$ — $(x + 4)(x - 5)$

25. Factor $n^2 + 5n - 24$. Show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3$, and 4.

$(n + 8)(n - 3)$

n	$n^2 + 5n - 24$
0	$0^2 + 5(0) - 24 = -24$
1	$1^2 + 5(1) - 24 = -18$
2	$2^2 + 5(2) - 24 = -10$
3	$3^2 + 5(3) - 24 = 0$
4	$4^2 + 5(4) - 24 = 12$

n	$(n + 8)(n - 3)$
0	$(0 + 8)(0 - 3) = -24$
1	$(1 + 8)(1 - 3) = -18$
2	$(2 + 8)(2 - 3) = -10$
3	$(3 + 8)(3 - 3) = 0$
4	$(4 + 8)(4 - 3) = 12$

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LESSON 8-3

Practice C

Factoring $x^2 + bx + c$

Factor each trinomial.

1. $x^2 + 10x + 24$ — $(x + 6)(x + 4)$
2. $y^2 + 12y + 20$ — $(y + 2)(y + 10)$
3. $a^2 + 15a + 54$ — $(a + 9)(a + 6)$
4. $h^2 + 18h + 45$ — $(h + 15)(h + 3)$
5. $x^2 + 16x + 48$ — $(x + 12)(x + 4)$
6. $c^2 + 15c + 50$ — $(c + 5)(c + 10)$
7. $x^2 - 16x + 48$ — $(x - 12)(x - 4)$
8. $d^2 - 19d + 88$ — $(d - 11)(d - 8)$
9. $x^2 - 20x + 36$ — $(x - 2)(x - 18)$
10. $m^2 - 43m + 42$ — $(m - 1)(m - 42)$
11. $x^2 - 16x + 28$ — $(x - 2)(x - 14)$
12. $n^2 - 12n + 35$ — $(n - 7)(n - 5)$
13. $f^2 + 3f - 28$ — $(f + 7)(f - 4)$
14. $b^2 + 11b - 42$ — $(b + 14)(b - 3)$
15. $x^2 + 12x - 160$ — $(x + 20)(x - 8)$
16. $g^2 + 2g - 48$ — $(g + 8)(g - 6)$
17. $k^2 + 16k - 36$ — $(k + 18)(k - 2)$
18. $x^2 + 2x - 63$ — $(x + 9)(x - 7)$
19. $p^2 - 2p - 8$ — $(p + 2)(p - 4)$
20. $x^2 - x - 72$ — $(x + 8)(x - 9)$
21. $q^2 - 3q - 18$ — $(q + 3)(q - 6)$
22. $x^2 - 4x - 32$ — $(x + 4)(x - 8)$
23. $t^2 - 10t - 39$ — $(t + 3)(t - 13)$
24. $w^2 - 20w - 125$ — $(w + 5)(w - 25)$

25. Factor $n^2 + 8n - 48$. Show that the original polynomial and the factored form describe the same sequence of numbers for $n = 0, 1, 2, 3$, and 4.

$(n + 12)(n - 4)$

n	$n^2 + 8n - 48$
0	$0^2 + 8(0) - 48 = -48$
1	$1^2 + 8(1) - 48 = -39$
2	$2^2 + 8(2) - 48 = -28$
3	$3^2 + 8(3) - 48 = -15$
4	$4^2 + 8(4) - 48 = 0$

n	$(n + 12)(n - 4)$
0	$(0 + 12)(0 - 4) = -48$
1	$(1 + 12)(1 - 4) = -39$
2	$(2 + 12)(2 - 4) = -28$
3	$(3 + 12)(3 - 4) = -15$
4	$(4 + 12)(4 - 4) = 0$

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LESSON 8-3

Reteach

Factoring $x^2 + bx + c$

When factoring $x^2 + bx + c$:

If c is positive	and b is positive	both factors are positive.
	and b is negative	both factors are negative.

Factor $x^2 + 7x + 10$. Check your answer.

$x^2 + 7x + 10$

Need factors of **10** that sum to **7**.

Factors of 10	Sum
1 and 10	11 ✗
2 and 5	7 ✓

$(x + 2)(x + 5)$

Check:

$(x + 2)(x + 5) = x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$ ✓

Factor $x^2 - 9x + 18$. Check your answer.

$x^2 - 9x + 18$

Need factors of **18** that sum to **−9**.

Factors of 18	Sum
−1 and −18	−19 ✗
−2 and −9	−11 ✗
−3 and −6	−9 ✓

$(x - 3)(x - 6)$

Check:

$(x - 3)(x - 6) = x^2 - 6x - 3x + 18$
$= x^2 - 9x + 18$ ✓

Factor the trinomial by filling in the blanks below.

1. $x^2 + 10x + 16$

Need factors of [16], that sum to [10].

Factors	Sum
[1] and [16]	[17]
[2] and [8]	[10]
[4] and [4]	[8]

$(x + 2)(x + 8)$

2. $x^2 - 9x + 20$

Need factors of [20], that sum to [−9].

Factors	Sum
[−1] and [−20]	[−21]
[−2] and [−10]	[−12]
[−3] and [−5]	[−9]

$(x - 4)(x - 5)$

Factor each trinomial.

3. $x^2 + 13x + 12$ — $(x + 12)(x + 1)$
4. $x^2 + 15x + 50$ — $(x + 10)(x + 5)$
5. $x^2 - 13x + 36$ — $(x - 9)(x - 4)$

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LESSON 8-3

Reteach

Factoring $x^2 + bx + c$ (continued)

When factoring $x^2 + bx + c$:

If c is negative	and b is positive	the larger factor must be positive.
	and b is negative	the larger factor must be negative.

Factor $x^2 + 8x - 20$. Check your answer.

$x^2 + 8x - 20$

Need factors of -20 that sum to 8. (Make larger factor positive.)

Factors of -20	Sum
-1 and 20	19 ✗
-2 and 10	8 ✓
-4 and 5	1 ✗

$(x - 2)(x + 10)$

Check:

$(x - 2)(x + 10) = x^2 + 10x - 2x - 20$
$= x^2 + 8x - 20$ ✓

Factor $x^2 - 3x - 28$. Check your answer.

$x^2 - 3x - 28$

Need factors of -28 that sum to -3. (Make larger factor negative.)

Factors of -28	Sum
1 and -28	-27 ✗
2 and -14	-12 ✗
4 and -7	-3 ✓

$(x + 4)(x - 7)$

Check:

$(x + 4)(x - 7) = x^2 - 7x + 4x - 28$
$= x^2 - 3x - 28$ ✓

Factor the trinomial by filling in the blanks below.

6. $x^2 + x - 20$

Need factors of $\boxed{-20}$, that sum to $\boxed{1}$.

Factors	Sum
$\boxed{-1}$ and $\boxed{20}$	$\boxed{19}$
$\boxed{-2}$ and $\boxed{10}$	$\boxed{8}$
$\boxed{-4}$ and $\boxed{5}$	$\boxed{1}$

$(x - 4)(x + 5)$

7. $x^2 - 3x - 4$

Need factors of $\boxed{-4}$, that sum to $\boxed{-3}$.

Factors	Sum
$\boxed{1}$ and $\boxed{-4}$	$\boxed{-3}$
$\boxed{2}$ and $\boxed{-2}$	$\boxed{0}$

$(x + 1)(x - 4)$

Factor each trinomial.

8. $x^2 + 3x - 18$ $(x - 3)(x + 6)$

9. $x^2 - 5x - 14$ $(x - 7)(x + 2)$

10. $x^2 + 4x - 45$ $(x - 5)(x + 9)$

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LESSON 8-3

Challenge

Pushing Back the Boundaries of Factoring

One day, Denise's mathematics teacher, Ms. Dolenze, asked her to factor $6x^2 + 5x + 1$. After puzzling over the problem for a while, Denise told Ms. Dolenze that the expression could not be factored. Amazed, Ms. Dolenze asked Denise why. "I could not find two binomials in x with integer coefficients whose product is $6x^2 + 5x + 1$," replied Denise. Then Ms. Dolenze quickly responded, "Why should that stop you?"

In the exercises that follow, you will see how to push back the boundaries of factoring.

In Exercise 1–4, answer each question in order to help Denise factor $6x^2 + 5x + 1$.

1. By what property of real numbers, can you justify $6x^2 + 5x + 1 = 6\left(x^2 + \frac{5}{6}x + \frac{1}{6}\right)$?

Distributive Property

2. Show that $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$ and that $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$; $\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \times \frac{3}{3} + \frac{1}{3} \times \frac{2}{2} = \frac{3 + 2}{2 \times 3} = \frac{5}{6}$

3. Show that $6\left(x + \frac{1}{2}\right)\left(x + \frac{1}{3}\right) = 6x^2 + 5x + 1$.

$6\left(x + \frac{1}{2}\right)\left(x + \frac{1}{3}\right) = 2\left(x + \frac{1}{2}\right) \times 3\left(x + \frac{1}{3}\right) = (2x + 1)(3x + 1)$
$= 6x^2 + 3x + 2x + 1 = 6x^2 + 5x + 1$

4. Is the product in Exercise 3 a factorization of $6x^2 + 5x + 1$? Is it necessary to find integers in order to factor a quadratic trinomial? Explain your response.

Yes; the given polynomial expression is written as a product of polynomials of lower degree. There is no requirement that the coefficients be integers.

Factor each quadratic trinomial as a number times a product of two linear binomials in x, each having rational numbers among the coefficients. Use Exercises 1–3 above as a model.

5. $15x^2 + 8x + 1$ $15\left(x + \frac{1}{5}\right)\left(x + \frac{1}{3}\right)$

6. $28x^2 + 11x + 1$ $28\left(x + \frac{1}{7}\right)\left(x + \frac{1}{4}\right)$

7. $6x^2 - 23x + 20$ $6\left(x - \frac{4}{3}\right)\left(x - \frac{5}{2}\right)$

8. $12x^2 + x - 35$ $12\left(x - \frac{5}{3}\right)\left(x + \frac{7}{4}\right)$

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LESSON 8-3

Problem Solving

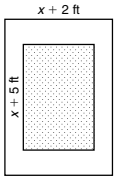
Factoring $x^2 + bx + c$

Write the correct answer.

1. A plot of land is rectangular and has an area of $x^2 - 5x - 24$ m^2. The length is $x + 3$ m. Find the width of the plot.

$x - 8$ m

2. An antique Persian carpet has an area of $(x^2 + x - 20)$ ft^2 and a length of $(x + 5)$ feet. The rug is displayed on a wall in a museum. The wall has a width of $(x + 2)$ feet and an area of $(x^2 + 17x + 30)$ ft^2. Write expressions for the length and width of both the rug and wall. Then find the dimensions of the rug and the wall if $x = 20$ feet.



rug: $(x + 5)(x - 4)$, wall: $(x + 2)(x + 15)$;
rug: 16 ft by 25 ft, wall: 22 ft by 35 ft

3. The area of a poster board is $x^2 + 3x - 10$ inches. The width is $x - 2$ inches.

a. Write an expression for the length of the poster board.

$x + 5$

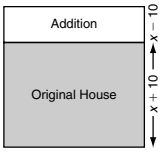
b. Find the dimensions of the poster board when $x = 14$.

12 in. by 19 in.

c. Write a polynomial for the area of the poster board if one inch is removed from each side.

$x^2 + x - 12$

The figure shows the plans for an addition on the back of a house. Use the figure to answer questions 4–6. Select the best answer.



4. The area of the addition is $(x^2 + 10x - 200)$ ft^2. What is its length?

A $(x - 20)$ feet
B $(x - 2)$ feet
C $(x + 2)$ feet
D $(x + 20)$ feet

5. What is the area of the original house?

F $(x^2 - 10x - 200)$ ft^2
G $(x^2 + 8x - 20)$ ft^2
H $(x^2 + 12x + 200)$ ft^2
J $(x^2 + 30x + 200)$ ft^2

6. The homeowners decide to extend the addition. The area with the addition is now $(x^2 + 12x - 160)$ ft^2. By how many feet was the addition extended?

A 1 foot **C** 3 feet
B 2 feet **D** 4 feet

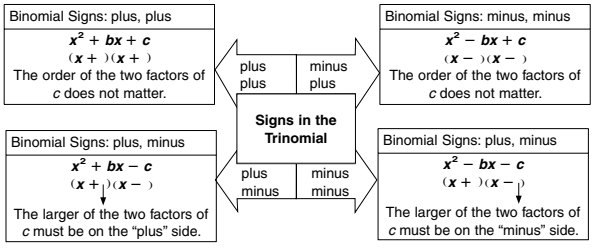
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LESSON 8-3

Reading Strategies

Use a Graphic Organizer

To factor trinomials of the form $x^2 + bx + c$ into two binomials, it is important to pay attention to the signs. Although you must figure out which factors of c will work, you can use the graphic organizer below to help you sort out the signs.



Fill in the signs of the two binomials that will form the correct factors of the given trinomial.

1. $x^2 + 9x + 18$ $(x \boxed{+} 6)(x \boxed{+} 3)$

2. $x^2 - 3x - 10$ $(x \boxed{+} 2)(x \boxed{-} 5)$

3. $x^2 + 13x + 36$ $(x \boxed{+} 9)(x \boxed{+} 4)$

4. $x^2 - 7x + 12$ $(x \boxed{-} 3)(x \boxed{-} 4)$

5. $x^2 + 3x - 4$ $(x \boxed{+} 4)(x \boxed{-} 1)$

6. $x^2 - 9x + 14$ $(x \boxed{-} 7)(x \boxed{-} 2)$

7. $x^2 + 5x - 50$ $(x \boxed{+} 10)(x \boxed{-} 5)$

8. $x^2 - x - 42$ $(x \boxed{+} 6)(x \boxed{-} 7)$

9. $x^2 + 6x + 9$ $(x \boxed{+} 3)(x \boxed{+} 3)$

Factor each trinomial into two binomials.

10. $x^2 - 10x + 21$ $(x - 3)(x - 7)$

11. $x^2 + 5x - 24$ $(x + 8)(x - 3)$

12. $x^2 + 11x + 30$ $(x + 6)(x + 5)$

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