LESSON 8-4

## Practice A

### Factoring $ax^2 + bx + c$

**Factor each trinomial, where $c$ is positive.**

1. $5x^2 + 17x + 6$ — $(1x + 3)(5x + 2)$
2. $4x^2 + 16x + 15$ — $(2x + 3)(2x + 5)$
3. $3x^2 + 17x + 20$ — $(3x + 6)(x + 4)$
4. $6x^2 + 19x + 10$ — $(3x + 2)(2x + 5)$
5. $8x^2 + 18x + 7$ — $(2x + 1)(4x + 7)$
6. $8x^2 + 19x + 3$ — $(2x + 3)(4x + 1)$
7. $4x^2 - 33x + 8$ — $(4x - 1)(1x - 8)$
8. $9x^2 - 27x + 14$ — $(3x - 7)(3x - 2)$
9. $6x^2 - 25x + 25$ — $(3x - 5)(2x - 5)$
10. $5x^2 - 22x + 8$ — $(5x - 2)(x - 4)$
11. $21x^2 - 22x + 5$ — $(7x - 5)(3x - 1)$
12. $12x^2 - 25x + 12$ — $(4x - 3)(3x - 4)$

**Factor each trinomial, where $c$ is negative.**

13. $10x^2 + 13x - 9$ — $(5x + 9)(2x - 1)$
14. $3x^2 + x - 4$ — $(3x + 4)(1x - 1)$
15. $5x^2 + 7x - 6$ — $(x + 2)(5x - 3)$
16. $4x^2 - 9x - 9$ — $(4x + 3)(x - 3)$
17. $4x^2 - 12x - 7$ — $(2x + 1)(2x - 7)$
18. $6x^2 - 7x - 20$ — $(3x + 4)(2x - 5)$

**Factor each trinomial, where $a$ is negative.**

19. $-5x^2 - 48x - 27$ — $-1(5x^2 + 48x + 27)$; $-1(5x + 3)(1x + 9)$
20. $-6x^2 + 11x - 4$ — $-1(2x - 1)(3x - 4)$
21. $-20x^2 + 7x + 6$ — $-1(5x + 2)(4x - 3)$

22. The area of a rectangle is $8x^2 + 14x + 3$. The length is $2x + 3$. The width is $4x + 1$.

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LESSON 8-4

## Practice B

### Factoring $ax^2 + bx + c$

**Factor each trinomial.**

1. $2x^2 + 13x + 15$ — $(2x + 3)(x + 5)$
2. $3x^2 + 10x + 8$ — $(3x + 4)(x + 2)$
3. $4x^2 + 24x + 27$ — $(2x + 9)(2x + 3)$
4. $5x^2 + 21x + 4$ — $(5x + 1)(x + 4)$
5. $4x^2 + 11x + 7$ — $(4x + 7)(x + 1)$
6. $6x^2 - 23x + 20$ — $(3x - 4)(2x - 5)$
7. $7x^2 - 59x + 24$ — $(7x - 3)(x - 8)$
8. $3x^2 - 14x + 15$ — $(3x - 5)(x - 3)$
9. $8x^2 - 73x + 9$ — $(8x - 1)(x - 9)$
10. $2x^2 + 11x - 13$ — $(2x + 13)(x - 1)$
11. $3x^2 + 2x - 16$ — $(3x + 8)(x - 2)$
12. $2x^2 + 17x - 30$ — $(x + 10)(2x - 3)$
13. $8x^2 + 29x - 12$ — $(x + 4)(8x - 3)$
14. $11x^2 + 25x - 24$ — $(x + 3)(11x - 8)$
15. $9x^2 - 3x - 2$ — $(3x + 1)(3x - 2)$
16. $12x^2 - 7x - 12$ — $(4x + 3)(3x - 4)$
17. $9x^2 - 49x - 30$ — $(9x + 5)(x - 6)$
18. $6x^2 + x - 40$ — $(3x + 8)(2x - 5)$
19. $-12x^2 - 35x - 18$ — $-1(4x + 9)(3x + 2)$
20. $-20x^2 + 29x - 6$ — $-1(5x - 6)(4x - 1)$
21. $-2x^2 + 5x + 42$ — $-1(2x + 7)(x - 6)$

22. The area of a rectangle is $20x^2 - 27x - 8$. The length is $4x + 1$. What is the width? $5x - 8$

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LESSON 8-4

## Practice C

### Factoring $ax^2 + bx + c$

**Factor each trinomial.**

1. $6d^2 + 17d + 7$ — $(3d + 7)(2d + 1)$
2. $14x^2 - 27x + 9$ — $(7x - 3)(2x - 3)$
3. $16c^2 + 18c + 5$ — $(2c + 1)(8c + 5)$
4. $20b^2 + 21b - 5$ — $(4b + 5)(5b - 1)$
5. $12m^2 - 40m + 25$ — $(6m - 5)(2m - 5)$
6. $6x^2 + 13x - 28$ — $(2x + 7)(3x - 4)$
7. $-20x^2 - 57x - 27$ — $-1(5x + 3)(4x + 9)$
8. $4t^2 + 21t - 49$ — $(t + 7)(4t - 7)$
9. $15f^2 + 37f + 18$ — $(5f + 9)(3f + 2)$
10. $64h^2 - 28h - 15$ — $(16h + 5)(4h - 3)$
11. $15a^2 - 68a + 32$ — $(15a - 8)(a - 4)$
12. $-4x^2 - 8x + 45$ — $-1(2x + 9)(2x - 5)$
13. $6x^2 + 91x - 150$ — $(3x + 50)(2x - 3)$
14. $4x^2 + 13x + 10$ — $(4x + 5)(x + 2)$
15. $9k^2 - 18k + 8$ — $(3k - 4)(3k - 2)$
16. $24n^2 + 17n - 22$ — $(8n + 11)(3n - 2)$
17. $21x^2 - 16x - 16$ — $(7x + 4)(3x - 4)$
18. $18p^2 + 9p - 14$ — $(6p + 7)(3p - 2)$
19. $-13w^2 + 38w - 25$ — $-1(13w - 25)(w - 1)$
20. $12x^2 + 77x + 30$ — $(12x + 5)(x + 6)$
21. $12y^2 - 41y - 15$ — $(3y + 1)(4y - 15)$

22. The area of a parallelogram is $32x^2 + 28x - 15$. The base is $8x - 3$. What is the height? $4x + 5$

23. The area of a towel is $6x^2 - 11x - 72$ inches. What is the width if the length is $3x + 8$ inches? $2x - 9$ inches

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LESSON 8-4

## Reteach

### Factoring $ax^2 + bx + c$

When factoring $ax^2 + bx + c$, first find factors of $a$ and $c$. Then check the products of the inner and outer terms to see if the sum is $b$.

**Factor $2x^2 + 11x + 15$. Check your answer.**

$2x^2 + 11x + 15 = (\square x + \square)(\square x + \square)$

| Factors of 2 | Factors of 15 | Outer + Inner |
|---|---|---|
| 1 and 2 | 1 and 15 | $1 \cdot 15 + 2 \cdot 1 = 17$ ✗ |
| 1 and 2 | 15 and 1 | $1 \cdot 1 + 2 \cdot 15 = 31$ ✗ |
| 1 and 2 | 5 and 3 | $1 \cdot 3 + 2 \cdot 5 = 13$ ✗ |
| 1 and 2 | 3 and 5 | $1 \cdot 5 + 2 \cdot 3 = 11$ ✓ |

$(x + 3)(2x + 5)$

***Check:***

$(x + 3)(2x + 5) = 2x^2 + 5x + 6x + 15$
$= 2x^2 + 11x + 15$ ✓

**Factor $3x^2 - 23x + 14$. Check your answer.**

$3x^2 - 23x + 14 = (\square x + \square)(\square x + \square)$

| Factors of 3 | Factors of 14 | Outer + Inner |
|---|---|---|
| 1 and 3 | −1 and −14 | $1 \cdot (-14) + 3 \cdot (-1) = -17$ ✗ |
| 1 and 3 | −14 and −1 | $1 \cdot (-1) + 3 \cdot (-14) = -42$ ✗ |
| 1 and 3 | −2 and −7 | $1 \cdot (-7) + 3 \cdot (-2) = -13$ ✗ |
| 1 and 3 | −7 and −2 | $1 \cdot (-2) + 3 \cdot (-7) = -23$ ✓ |

$(x - 7)(3x - 2)$

***Check:***

$(x - 7)(3x - 2) = 3x^2 - 2x - 21x + 14$
$= 3x^2 - 23x + 14$ ✓

1. Factor $5x^2 + 12x + 4$ by filling in the blanks below.

| Factors | Factors | Outer + Inner |
|---|---|---|
| 1 and 5 | 1 and 4 | $1 \cdot 4 + 5 \cdot 1 = 9$ |
| 1 and 5 | 4 and 1 | $1 \cdot 1 + 5 \cdot 4 = 21$ |
| 1 and 5 | 2 and 2 | $1 \cdot 2 + 5 \cdot 2 = 12$ |

$(x + 2)(5x + 2)$

**Factor each trinomial.**

2. $3x^2 + 7x + 4$ — $(3x + 4)(x + 1)$
3. $2x^2 - 13x + 21$ — $(2x - 7)(x - 3)$
4. $4x^2 + 8x + 3$ — $(2x + 3)(2x + 1)$

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LESSON 8-4

## Reteach

### *Factoring $ax^2 + bx + c$ (continued)*

When $c$ is negative, one factor of $c$ is positive and one is negative. You can stop checking factors when you find the factors that work.

**Factor $2x^2 + 7x - 15$. Check your answer.**

$2x^2 + 7x - 15 = (\square x + \square)(\square x + \square)$

| Factors of 2 | Factors of −15 | Outer + Inner |
|---|---|---|
| 1 and 2 | −3 and 5 | $1 \cdot 5 + 2 \cdot (-3) = -1$ *x* |
| 1 and 2 | 3 and −5 | $1 \cdot (-5) + 2 \cdot 3 = 1$ *x* |
| 1 and 2 | −5 and 3 | $1 \cdot 3 + 2 \cdot (-5) = -7$ *x* |
| 1 and 2 | 5 and −3 | $1 \cdot (-3) + 2 \cdot 5 = 7$ ✓ |

$(x + 5)(2x - 3)$

***Check:***

$(x + 5)(2x - 3) = 2x^2 - 3x + 10x - 15$
$= 2x^2 + 7x - 15$

When $a$ is negative, factor out $-1$. Then factor as shown previously.

**Factor $-5x^2 + 28x + 12$. Check your answer.**

$-5x^2 + 28x + 12$

$-1(5x^2 - 28x - 12) = -1(\square x + \square)(\square x + \square)$

| Factors of 5 | Factors of −12 | Outer + Inner |
|---|---|---|
| 1 and 5 | −2 and 6 | $1 \cdot 6 + 5 \cdot (-2) = -4$ *x* |
| 1 and 5 | 2 and −6 | $1 \cdot (-6) + 5 \cdot 2 = 4$ *x* |
| 1 and 5 | 6 and −2 | $1 \cdot (-2) + 5 \cdot 6 = 28$ *x* |
| 1 and 5 | −6 and 2 | $1 \cdot 2 + 5 \cdot (-6) = -28$ ✓ |

$-1(x - 6)(5x + 2)$

***Check:***

$-1(x - 6)(5x + 2)$
$= -1(5x^2 + 2x - 30x - 12)$
$= -1(5x^2 - 28x - 12)$
$= -5x^2 + 28x + 12$

**Factor each trinomial.**

**5.** $3x^2 - 7x - 20$ — $(3x + 5)(x - 4)$

**6.** $5x^2 + 34x - 7$ — $(5x - 1)(x + 7)$

**7.** $-2x^2 + 3x + 5$ — $-1(2x - 5)(x + 1)$

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LESSON 8-4

## Challenge

### *Finding Factors by Synthetic Division*

**Synthetic division** is a quick way to divide a polynomial by a binomial in the form $x - a$ yielding another factor of the polynomial.

**$(x - 5)$ is a factor of $2x^2 - 3x - 35$. Use synthetic division to find the other factor.**

$x - 5$ | $2x^2 - 3x - 35$

| 5 | 2 | −3 | −35 |
|---|---|---|---|
| | | 10 | 35 |
| | 2 | 7 | 0 |

The other factor is $2x + 7$.

**Step 1:** Write the coefficients of the polynomial (in standard form). Write $a$ in a box to the left. Draw a line below the coefficients.

**Step 2:** Write the first coefficient below the line.

**Step 3:** Multiply that coefficient by the value of $a$ and write the product under the next coefficient.

**Step 4:** Add the numbers in the second column.

**Step 5:** Repeat until all additions are completed.

If done correctly, the last sum will be zero. The numbers under the bar are the coefficients of the quotient, in decreasing degree.

**A polynomial and one factor is given. Use synthetic division to find the other factor.**

**1.** $4x^2 - 7x - 15$; $(x - 3)$

| 3 | 4 | −7 | −15 |
|---|---|---|---|
| | | 12 | 15 |
| | 4 | 5 | 0 |

$(4x + 5)$

**2.** $3x^2 + 4x - 32$; $(x + 4)$

| −4 | 3 | 4 | −32 |
|---|---|---|---|
| | | −12 | 32 |
| | 3 | −8 | 0 |

$(3x - 8)$

**3.** $6x^2 - 29x - 5$; $(x - 5)$ — $(6x + 1)$

**4.** $8x^2 + 11x - 10$; $(x + 2)$ — $(8x - 5)$

Some polynomials have three factors.

**$(x - 2)$ is a factor of $x^3 - 3x^2 - 10x + 24$. Factor completely.**

| 2 | 1 | −3 | −10 | 24 |
|---|---|---|---|---|
| | | 2 | −2 | −24 |
| | 1 | −1 | −12 | 0 |

Now factor $x^2 - x - 12$ using a method you know.
$x^2 - x - 12 = (x + 3)(x - 4)$

The factors are: $(x - 2)(x + 3)(x - 4)$.

**A polynomial and one factor is given. Factor completely.**

**5.** $x^3 + 7x^2 + 7x - 15$; $(x - 1)$ — $(x - 1)(x + 3)(x + 5)$

**6.** $2x^3 - 3x^2 - 39x + 20$; $(x + 4)$ — $(x + 4)(2x - 1)(x - 5)$

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LESSON 8-4

## Problem Solving

### *Factoring $ax^2 + bx + c$*

**Write the correct answer.**

**1.** A rectangular painting has an area of $(2x^2 + 8x + 6)$ cm$^2$. Its length is $(2x + 2)$ cm. Find the width of the painting.

$(x + 3)$ cm

**2.** A ball is kicked straight up into the air. The height of the ball in feet is given by the expression $-16t^2 + 12t + 4$, where $t$ is time in seconds. Factor the expression. Then find the height of the ball after 1 second.

$-1(4t - 4)(4t + 1)$ or $-4(4t + 1)(t - 1)$; 0 feet

**3.** Instructors led an exercise class from a raised rectangular platform at the front of the room. The width of the platform was $(3x - 1)$ feet and the area was $(9x^2 + 6x - 3)$ ft$^2$. Find the length of this platform. After the exercise studio is remodeled, the area of the platform will be $(9x^2 + 12x + 3)$ ft$^2$. By how many feet will the width of the platform change?

$(3x + 3)$ ft; increase of 2 ft

**4.** A clothing store has a rectangular clearance section with a length that is twice the width $w$. During a sale, the section is expanded to an area of $(2w^2 + 19w + 35)$ ft$^2$. Find the amount of the increase in the length and width of the clearance section.

length increased by 5 ft, width increased by 7 ft

**Select the best answer.**

**5.** The area of a soccer field is $(24x^2 + 100x + 100)$ m$^2$. The width of the field is $(4x + 10)$ m. What is the length?

**A** $(3x + 10)$ m **C** $(6x + 10)$ m
**B** $(6x + 1)$ m **D** $(8x + 2)$ m

**6.** A square parking lot has an area of $(4x^2 + 20x + 25)$ ft$^2$. What is the length of one side of the parking lot?

**F** $(2x + 5)$ ft **H** $(5x + 4)$ ft
**G** $(2x + 10)$ ft **J** $(5x + 2)$ ft

**7.** For a certain college, the number of applications received after $x$ recruiting seminars is modeled by the polynomial $3x^2 + 490x + 6000$. What is this expression in factored form?

**A** $(3x - 40)(x - 150)$
**B** $(3x + 40)(x + 150)$
**C** $(3x - 30)(x - 200)$
**D** $(3x + 30)(x + 200)$

**8.** Jin needs to fence in his rectangular backyard. The fence will have one long section away from, but parallel to, the length of his house and two shorter sides connecting that section to the house. The length of Jin's house is $(3x + 4)$ yd and the area of his backyard is $(9x^2 + 15x + 4)$ yd$^2$. How many yards of fencing will Jin need?

**F** $(6x + 2)$ yd **H** $(9x + 9)$ yd
**G** $(9x + 6)$ yd **J** $(12x + 10)$ yd

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LESSON 8-4

## Reading Strategies

### *Use a Model*

Most people factor trinomials of the form $ax^2 + bx + c$ using a sort of organized guess-and-check method. Look at the model below.

Factor the trinomial $4x^2 - 17x - 15$ into two binomials.

- **Recall:** The acronym FOIL stands for First, Outer, Inner, Last.
- **Signs:** This trinomial requires a "plus" and a "minus" binomial.
- **Setup:** (F + L)(F − L) → $(\square x + \square)(\square x - \square)$
- **"First":** The factorizations of 4 are: $1 \cdot 4$ and $2 \cdot 2$.
- **"Last":** The factorizations of 15 are: $1 \cdot 15$ and $3 \cdot 5$.
- **"Outer" and "Inner":** Guess-and-check these values in the binomials. The goal is to multiply the "Outer" terms and the "Inner" terms, and get a sum of $-17$.

$(\downarrow x + \downarrow)(\downarrow x - \downarrow)$ (O, I)

| | | | | Outer + Inner | |
|---|---|---|---|---|---|
| 1 | 1 | 4 | −15 | $1 \cdot -15 + 1 \cdot 4 = -11$ | No |
| 1 | 15 | 4 | −1 | $1 \cdot -1 + 15 \cdot 4 = 59$ | No |
| 1 | 5 | 4 | −3 | $1 \cdot -3 + 5 \cdot 4 = -7$ | No |
| 1 | 3 | 4 | −5 | $1 \cdot -5 + 3 \cdot 4 = 7$ | No |
| 4 | 1 | 1 | −15 | $4 \cdot -15 + 1 \cdot 1 = -59$ | No |
| 4 | 15 | 1 | −1 | $4 \cdot -1 + 15 \cdot 1 = 11$ | No |
| 4 | 5 | 1 | −3 | $4 \cdot -3 + 5 \cdot 1 = -7$ | No |
| **4** | **3** | **1** | **−5** | $4 \cdot -5 + 3 \cdot 1 = -17$ | Yes |

- **Answer:** $4x^2 - 17x - 15$ factors into $(4x + 3)(1x - 5)$, or just $(4x + 3)(x - 5)$.

**Complete the following to guide you in factoring $3x^2 - 20x + 12$.**

**1.** What are the factorizations of 3? $1 \cdot 3$

**2.** What are the factorizations of 12? $1 \cdot 12; 2 \cdot 6; 3 \cdot 4$

**3.** What signs are needed in each of the two binomials? minus, minus

**4.** Set up the binomials for factoring this polynomial. $(x - )(x - )$

**5.** Guess-and-check the factors that will result in the correct middle term coefficient of 20.

Possible answer:

$(\downarrow x - \downarrow)(\downarrow x - \downarrow)$

| | | | Outer + Inner | |
|---|---|---|---|---|
| 3 | −1 | 1 | −12 | $3 \cdot -12 + -1 \cdot 1 = -37$ | No |
| 3 | −12 | 1 | −1 | $3 \cdot -1 + -12 \cdot 1 = -15$ | No |
| 3 | −4 | 1 | −3 | $3 \cdot -3 + -4 \cdot 1 = -13$ | No |
| 3 | −3 | 1 | −4 | $3 \cdot -4 + -3 \cdot 1 = -15$ | No |
| 3 | −6 | 1 | −2 | $3 \cdot -2 + -6 \cdot 1 = -12$ | No |
| 3 | −2 | 1 | −6 | $3 \cdot -6 + -2 \cdot 1 = -20$ | Yes |

**6.** What is the final answer for factoring $3x^2 - 20x + 12$? $(3x - 2)(x - 6)$

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