LESSON 8-5

Practice A

Factoring Special Products

Factor each perfect square trinomial by filling in the blanks.

1. $x^2 + 10x + 25 = (x + 5)(x + \boxed{5}) = (\boxed{x} + \boxed{5})^2$

   $x \cdot x$ $\quad 2(x \cdot 5)$ $\quad 5 \cdot 5$

2. $9x^2 + 6x + 1 = (3x + \boxed{1})(\boxed{3x} + 1) = (\boxed{3x} + \boxed{1})^2$

   $\boxed{3x} \cdot 3x$ $\quad \boxed{2}(3x \cdot 1)$ $\quad 1 \cdot \boxed{1}$

Factor each perfect square trinomial.

3. $x^2 - 18x + 81$ $(x - 9)^2$

4. $36x^2 + 24x + 4$ $(6x + 2)^2$

Complete the following sentences.

5. $x^2 + 6x + 6$ is not a perfect square trinomial because

   6 is not a perfect square.

6. $4x^2 + 12x + 36$ is not a perfect square trinomial because

   $12x \neq 2(2x \cdot 6)$.

7. A square floor tile has an area of $(x^2 + 8x + 16)$ in^2. The side length of the tile is of the form $cx + d$, where c and d are whole numbers.

   a. Find an expression for the side length of the tile. $x + 4$ in.

   b. Find an expression for the perimeter of the tile. $4(x + 4)$ in.

   c. Find the perimeter when $x = 8$ in. 48 in.

Factor each binomial into the difference of two squares.

8. $x^2 - 9 = (x + \boxed{3})(x - \boxed{3})$

9. $4p^2 - 49 = (\boxed{2p} + \boxed{7})(\boxed{2p} - \boxed{7})$

10. $t^6 - 144$ $(t^3 + 12)(t^3 - 12)$

11. $16x^{10} - y^2$ $(4x^5 + y)(4x^5 - y)$

Complete the following sentences.

12. $25n^2 - 20$ is not a difference of squares because

    20 is not a perfect square.

13. $9m^4 + 1$ is not a difference of squares because

    the operation between the two squares is addition.

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LESSON 8-5

Practice B

Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $x^2 + 6x + 9$

   yes; $(x + 3)^2$

2. $4x^2 + 20x + 25$

   yes; $(2x + 5)^2$

3. $36x^2 - 24x + 16$

   no; $24x \neq 2(6x \cdot 4)$

4. $9x^2 - 12x + 4$

   yes; $(3x - 2)^2$

5. A rectangular fountain in the center of a shopping mall has an area of $(4x^2 + 12x + 9)$ ft^2. The dimensions of the fountain are of the form $cx + d$, where c and d are whole numbers. Find an expression for the perimeter of the fountain. Find the perimeter when $x = 2$ ft.

   $4(2x + 3)$ ft; 28 ft

Determine whether each binomial is the difference of two squares. If so, factor it. If not, explain why.

6. $x^2 - 16$

   yes; $(x + 4)(x - 4)$

7. $9b^4 - 200$

   no; 200 is not a perfect square.

8. $1 - m^6$

   yes; $(1 + m^3)(1 - m^3)$

9. $36s^2 - 4t^2$

   yes; $(6s + 2t)(6s - 2t)$

10. $x^2y^2 + 196$

    no; the operation between the two squares is addition.

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LESSON 8-5

Practice C

Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $16x^2 + 72x + 81$

   yes; $(4x + 9)^2$

2. $x^2 - 14x - 49$

   no; the last term must be positive.

3. $x^2 - 2x + 1$

   yes; $(x - 1)^2$

4. $x^6 + 16x^3 + 64$

   yes; $(x^3 + 8)^2$

5. The area of a rectangular frame for Ken's artwork is given by $(25x^2 - 20x + 4)$ cm^2. The dimensions of the frame are of the form $cx + d$, where c and d are whole numbers. Find an expression for the perimeter of the frame. Find the perimeter when $x = 13$ cm.

   $4(5x - 2)$ cm; 252 cm

Determine whether each binomial is the difference of two squares. If so, factor it. If not, explain why.

6. $9y^2 - 121$

   yes; $(3y + 11)(3y - 11)$

7. $49 - t^6$

   yes; $(7 + t^3)(7 - t^3)$

8. $d^9 - 25$

   no; d^9 is not a perfect square.

9. $16p^4 - 100q^2$

   yes; $(4p^2 + 10q)(4p^2 - 10q)$

10. $x^4y^{10} + 324$

    no; the operation between the two squares is addition.

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LESSON 8-5

Reteach

Factoring Special Products

If a polynomial is a perfect square trinomial, the polynomial can be factored using a pattern.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Determine whether $4x^2 + 20x + 25$ is a perfect square trinomial. If so, factor it. If not, explain why.

Step 1: Find a, b, then $2ab$.

$a = \sqrt{4x^2} = 2x$	*The first term is a perfect square.*
$b = \sqrt{25} = 5$	*The last term is a perfect square.*
$2ab = 2(2x)(5) = 20x$	*Middle term* $(20x) = 2ab$.

Therefore, $4x^2 + 20x + 25$ is a perfect square trinomial.

Step 2: Substitute expressions for a and b into $(a + b)^2$.

$(2x + 5)^2$

Determine whether $9x^2 + 25x + 36$ is a perfect square trinomial. If so, factor it. If not, explain why.

Step 1: Find a, b, then $2ab$.

$a = \sqrt{9x^2} = 3x$	*The first term is a perfect square.*
$b = \sqrt{36} = 6$	*The last term is a perfect square.*
$2ab = 2(3x)(6) = 36x$	*Middle term* $(25x) \neq 2ab$.

STOP

Because $25x$ does not equal $2ab$, $9x^2 - 25x + 36$ is not a perfect square trinomial.

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $9x^2 + 30x + 100$	2. $x^2 - 14x + 49$	3. $25x^2 + 20x + 4$
$a =$ 3x	$a =$ x	$a =$ 5x
$b =$ 10	$b =$ 7	$b =$ 2
$2ab =$ 60x	$2ab =$ 14x	$2ab =$ 20x
Factor or explain:	Factor or explain:	Factor or explain:
$60x \neq 2ab$	$(x - 7)^2$	$(5x + 2)^2$

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LESSON 8-5 Reteach

Factoring Special Products (continued)

If a binomial is a difference of squares, it can be factored using a pattern.

$$a^2 - b^2 = (a + b)(a - b)$$

Determine whether $64x^2 - 25$ is a difference of squares. If so, factor it. If not, explain why.

Step 1: Determine if the binomial is a difference.

$64x^2 - 25$ *The minus sign indicates it is a difference.*

Step 2: Find a and b.

$a = \sqrt{64x^2} = 8x$ *The first term is a perfect square.*

$b = \sqrt{25} = 5$ *The last term is a perfect square.*

Therefore, $64x^2 - 25$ is a difference of squares.

Step 3: Substitute expressions for a and b into $(a + b)(a - b)$.

$(8x + 5)(8x - 5)$

Determine whether $4x^2 + 25$ is a difference of squares. If so, factor it. If not, explain why.

Step 1: Determine if the binomial is a difference.

$4x^2 + 25$ *The plus sign indicates a sum.*

STOP. The binomial is not a difference, so it cannot be a difference of squares. It does not have a GCF either, so $4x^2 + 25$ cannot be factored.

Determine whether each binomial is a difference of squares. If so, factor it. If not, explain why.

4. $25x^2 - 81$

Difference? yes

$a =$ $5x$

$b =$ 9

Factor or explain: $(5x + 9)(5x - 9)$

5. $30x^2 - 49$

Difference? yes

$a =$ $\sqrt{30}x$

$b =$ 7

Factor or explain: a is not a perfect square

6. $4x^2 - 121$

Difference? yes

$a =$ $2x$

$b =$ 11

Factor or explain: $(2x + 11)(2x - 11)$

Factor.

7. $x^2 - 100$ $(x + 10)(x - 10)$

8. $x^2 - y^2$ $(x + y)(x - y)$

9. $9x^4 - 64$ $(3x^2 + 8)(3x^2 - 8)$

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LESSON 8-5 Challenge

Exploring $a^n - b^n$ and $a^n + b^n$

You already know the facts stated below.

$a^2 - b^2 = (a + b)(a - b)$ and $a^2 + b^2$ cannot be factored by using real numbers.

In the exercises that follow, you will be able to study the factorability of differences and sums involving exponents greater than or equal to 2.

In Exercises 1 and 2, consider $a^n - b^n$ for $n = 3$ and $n = 4$, respectively.

1. Show that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ by performing the multiplication started at right. Show your work in the space provided.

$$\begin{array}{r} a^2 + ab + b^2 \\ \times \quad a - b \\ \hline -ba^2 - ab^2 - b^3 \\ +a^3 + ba^2 + ab^2 \\ \hline a^3 \qquad -b^3 \end{array}$$

2. Show that $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$ by following the reasoning started at right. Notice that the difference of fourth powers is written as a new difference of squares.

$$a^4 - b^4 = (a^2)^2 - (b^2)^2$$
$$= \left(\boxed{a^2} + \boxed{b^2}\right)\left(\boxed{a^2} - \boxed{b^2}\right)$$
$$= a^2 + b^2\,(a + b)\,(a - b)$$

In Exercises 3 and 4, consider $a^n + b^n$ for $n = 3$ and $n = 2$, respectively.

3. Show that $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ by performing the multiplication started at right. Show your work in the space provided.

$$\begin{array}{r} a^2 - ab + b^2 \\ \times \quad a + b \\ \hline ba^2 - ab^2 + b^3 \\ +a^3 - ba^2 + ab^2 \\ \hline a^3 \qquad +b^3 \end{array}$$

4. Suppose that $a^2 + b^2$ can be factored as $a^2 + b^2 = (ra + sb)(ta + bu)$, where r, s, t, and u are numbers and a and b are variables. The reasoning shown at right will help you see that $a^2 + b^2$ cannot be factored.

$$a^2 + b^2 = (ra + sb)(ta + bu)$$
$$= (rt)a^2 + (st + ru)ab + (su)b^2$$

So, $1 = rt$, $0 = st + ru$, and $1 = su$

$r = \frac{1}{t}$, $-st = ru$, and $s = \frac{1}{u}$

Thus, $-\frac{t}{u} = \frac{u}{t}$

$-t^2 = u^2$

$t = 0$ and $u = 0$

a. How do you know that $1 = rt$, $0 = st + ru$, and $1 = su$?

If two polynomials are equal, their corresponding coefficients are equal. The coefficients of a^2, ab, and b^2 are equal.

b. Explain how $t = 0$ and $u = 0$ tell you that there are no values of r and s that make the factorization possible.

If $t = 0$ and $u = 0$, then r and s are given by undefined expressions. Thus, there are no numbers r, s, t, and u for which $a^2 + b^2$ can be factored as $(ra + sb)(ta + bu)$.

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LESSON 8-5 Problem Solving

Factoring Special Products

Write the correct answer.

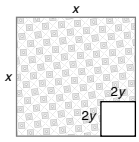
1. A rectangular fountain has an area of $(16x^2 + 8x + 1)$ ft^2. The dimensions of the rectangle have the form $ax + b$, where a and b are whole numbers. Write an expression for the perimeter of the fountain. Then find the perimeter when $x = 2$ feet.

$16x + 4$; 36 feet

2. A square tabletop has an area of $(9x^2 - 90x + 225)$ cm^2. The dimensions of the tabletop have the form $cx - d$, where c and d are whole numbers. Write an expression for the perimeter of the tabletop. Then find the perimeter when $x = 25$ centimeters.

$12x - 60$; 240 cm

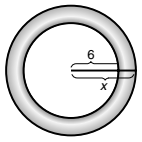
3. The floor plan of a daycare center is shown.



The arts and crafts area in the lower right corner is not carpeted. The rest of the center is carpeted. Write an expression, in factored form, for the area of the floor that is carpeted.

$(x + 2y)(x - 2y)$

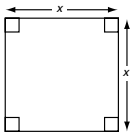
4. A plate with a decorative border is shown.



Write an expression, in factored form, for the area of the border. (*Hint:* First factor out the GCF.)

$\pi(x + 6)(x - 6)$

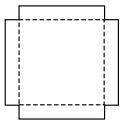
Nelson is making open top boxes by cutting out corners from a sheet of cardboard, folding the edges up, and then taping them together. Select the best answer.



5. Nelson cut corners so that each corner was a square with side lengths of 4. What is the total area of the remaining piece of cardboard?

A $x^2 - 8x + 16$ **C** $x^2 - 16x + 64$

B $x^2 + 8x + 16$ **D** $x^2 + 16x + 64$



6. What are the dimensions of the square corners if the total remaining area is $x^2 - 4x + 4$?

F 1 by 1 **H** 4 by 4

G 2 by 2 **J** 8 by 8

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LESSON 8-5 Reading Strategies

Compare and Contrast

The chart below shows how to recognize and factor two special products.

	Perfect Square Trinomial Ex: $x^2 + 6x + 9$	Contrast	Difference of Squares Ex: $4x^2 - 25$
Number of Terms	3	Number of Terms	2
Sign(s)	+ + or − +	Sign(s)	always −
Pattern	$x^2 + 6x + 9$ $x \cdot x$ $2(x \cdot 3)$ $3 \cdot 3$	Pattern	$4x^2 - 25$ $2x \cdot 2x$ $5 \cdot 5$
Factored Form	binomial squared $(x + 3)(x + 3)$ $= (x + 3)^2$	Factored Form	two binomials, opposite signs $(2x + 5)(2x - 5)$
		Compare	
	The first term and last term of both special products are perfect squares.		

Identify each of the following as a perfect square trinomial (P), a difference of squares (D), or neither (N).

1. $x^2 - 16$ D

2. $x^2 - 14x + 49$ P

3. $100x^2 - 36$ D

4. $4x^2 - 12x - 9$ N

5. $x^2 + 14x + 49$ P

6. $25x^2 + 64$ N

Factor each special product.

7. $16x^2 - 9$ $(4x + 3)(4x - 3)$

8. $121x^2 - 44x + 4$ $(11x - 2)^2$

9. $81x^2 - 1$ $(9x + 1)(9x - 1)$

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