LESSON 8-6

Practice A

Choosing a Factoring Method

Tell whether each polynomial is completely factored. If not, factor it.

1. $3(b^3 - 5)$ — yes
2. $2(n^3 + 4n^2)$ — no; $2n^2(n + 4)$
3. $8y(y^2 + 1)$ — yes
4. $(t - 6)(3t + 5)$ — yes
5. $2(m^2 + 10m + 9)$ — no; $2(m + 9)(m + 1)$
6. $(2p - 8)(p + 3)$ — no; $2(p - 4)(p + 3)$

Factor out a GCF. Then continue to factor by using other methods.

7. $45g^2 - 20$ — $5(3g + 2)(3g - 2)$
8. $3w^3 + 30w^2 + 75w$ — $3w(w + 5)^2$
9. $12x^2y - 48xy + 48y$ — $12y(x - 2)^2$
10. $-3d^3 + 300d$ — $-3d(d - 10)(d + 10)$
11. $2a^2 - 32$ — $2(a + 4)(a - 4)$
12. $5m^2 - 5m - 60$ — $5(m + 3)(m - 4)$

Factor completely.

13. $c^2 + 7c - 18$ — $(c + 9)(c - 2)$
14. $2x^2 + 8x + 6$ — $2(x + 3)(x + 1)$
15. $f^3 - 3f^2 + 4f - 12$ — $(f^2 + 4)(f - 3)$
16. $-6k^2 + 39k - 18$ — $-3(k - 6)(2k - 1)$
17. $p^8 - m^4$ — $(p^4 + m^2)(p^2 + m)(p^2 - m)$
18. $2a(a^2 - 1) + 7(a^2 - 1)$ — $(a + 1)(a - 1)(2a + 7)$

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LESSON 8-6

Practice B

Choosing a Factoring Method

Tell whether each polynomial is completely factored. If not, factor it.

1. $6(t^2 + 12)$ — yes
2. $5(m^2 + 9m)$ — no; $5m(m + 9)$
3. $2p(p^4 - 9)$ — no; $2p(p^2 + 3)(p^2 - 3)$
4. $(x - 8)(2x + 3)$ — yes
5. $3k^3(5k^2 + 19)$ — yes
6. $7(14g^4 - 4g + 10)$ — no; $14(7g^4 - 2g + 5)$

Factor each polynomial completely.

7. $24x + 40$ — $8(3x + 5)$
8. $5r^3 - 10r$ — $5r(r^2 - 2)$
9. $3x^3y + x^2y^2$ — $x^2y(3x + y)$
10. $-3a^2b + 12ab - 12b$ — $-3b(a - 2)^2$
11. $5t^3 - 45t + 3t^2 - 27$ — $(5t + 3)(t + 3)(t - 3)$
12. $2y^2 - 6y - 56$ — $2(y + 4)(y - 7)$
13. $6a^3 + 39a^2 + 45a$ — $3a(2a + 3)(a + 5)$
14. $x^3 - 9x$ — $x(x - 3)(x + 3)$
15. $12n^3 - 48$ — $12(n^3 - 4)$
16. $3c^4 + 24c^3 + 48c^2$ — $3c^2(c + 4)^2$
17. $3d^3 + 4d - 2$ — unfactorable
18. $10w^6 - 160w^2$ — $10w^2(w^2 + 4)(w - 2)(w + 2)$

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LESSON 8-6

Practice C

Choosing a Factoring Method

Tell whether each polynomial is completely factored. If not, factor it.

1. $4(n^4 - 8)$ — yes
2. $12h^2(g^2 + 6gh)$ — no; $12h^2g(g + 6h)$
3. $7(t^2 + 16)$ — yes
4. $(y + 9)(4y - 3)$ — yes
5. $5x^4(12x^3 - 6x^2 - 18x)$ — no; $30x^5(2x - 3)(x + 1)$
6. $(6b + 3)(b - 5)$ — no; $3(2b + 1)(b - 5)$

Factor each polynomial completely.

7. $-2r + 16$ — $-2(r - 8)$
8. $60c + 45c^2$ — $15c(4 + 3c)$
9. $24xy^5 + 9y^4$ — $3y^4(8xy + 3)$
10. $10k^3 - 80k^2 + 160k$ — $10k(k - 4)^2$
11. $3a^2 + 9a - 54$ — $3(a + 6)(a - 3)$
12. $5t^2 + 2t + 3$ — unfactorable
13. $-4f^3 - 14f^2 + 8f$ — $-2f(2f - 1)(f + 4)$
14. $m^5 - 25m^3$ — $m^3(m + 5)(m - 5)$
15. $8w^4 + 96w^2 + 288$ — $8(w^2 + 6)^2$
16. $6h^4 - 5h + 9$ — unfactorable
17. $4d^6 - 20d^5 + 12d^4 - 60d^3$ — $4d^3(d^2 + 3)(d - 5)$
18. $m^8n^2 - n^{18} - m^8 + n^{16}$ — $(m^4 + n^8)(m^2 + n^4)(m + n^2)(m - n^2)(n + 1)(n - 1)$

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LESSON 8-6

Reteach

Choosing a Factoring Method

Use the following table to help you choose a factoring method.

First factor out a GCF if possible. Then,

If binomial,	check for difference of squares.	yes →	Use $(a + b)(a - b)$.
		no →	If no GCF, it cannot be factored.
If trinomial,	check for perfect square trinomial.	yes →	Factor using $(a + b)^2$ or $(a - b)^2$.
		no →	If $a = 1$, check factors of c that sum to b.
			If $a \neq 1$, check inner plus outer factors of a and c that sum to b.
If 4 or more terms,	→		Try to factor by grouping.

Explain how to choose a factoring method for $x^2 - x - 30$. Then state the method.

- There is no GCF.
- $x^2 - x - 30$ is a trinomial.
- The terms a and b are not perfect squares, therefore this is not a perfect square trinomial.
- $a = 1$

Method: Factor by checking factors of c that sum to b.

Explain how to choose a factoring method for $2x^2 - 50$. Then state the method.

- Factor out the GCF: $2(x^2 - 25)$
- $x^2 - 25$ is a binomial.
- a and b are perfect squares. This is a difference of squares.

Method: Factor out GCF. Then use $(a + b)(a - b)$.

Explain how to choose a factoring method for each polynomial. Then state the method.

1. $x^2 + 14x + 49$ — no GCF; $x^2 + 14x + 49$ is a trinomial; This is a perfect square trinomial. Method: use $(a + b)^2$.
2. $4x^2 - 40$ — factor out the GCF: $4(x^2 - 10)$; $x^2 - 10$ is a binomial; This is not a difference of squares. Method: Factor out GCF.
3. $2x^2 + 8x + 6$ — factor out the GCF: $2(x^2 + 4x + 3)$; $x^2 + 4x + 3$ is a trinomial; This is not a perfect square trinomial; $a = 1$; Method: Factor out GCF. Then find factors of c that sum to b.

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LESSON 8-6

Reteach

Choosing a Factoring Method (continued)

It is often necessary to use more than one factoring method to factor a polynomial completely.

Factor $5x^2 - 5x - 60$ completely. Check your answer.

Step 1: Factor out the GCF.

$5x^2 - 5x - 60$

$5(x^2 - x - 12)$

Step 2: Choose a method for factoring.

- $x^2 - x - 12$ is a trinomial.
- It is not a perfect square.

Method: Find factors of c that will sum to b.

Step 3: Factor.

Factors of −12	Sum
2 and −6	−4 ✗
3 and −4	−1 ✓

$(x + 3)(x - 4)$

Step 4: Write the complete factorization.

$5(x + 3)(x - 4)$

Check:

$5(x + 3)(x - 4) = 5(x^2 - 4x + 3x - 12)$

$= 5(x^2 - x - 12)$

$= 5x^2 - 5x - 60$ ✓

Factor $16x^2 - 36$ completely. Check your answer.

Step 1: Factor out the GCF.

$16x^2 - 36$

$4(4x^2 - 9)$

Step 2: Choose a method for factoring.

- $4x^2 - 9$ is a binomial.
- It is a difference of squares.

Method: Use $(a + b)(a - b)$.

Step 3: Factor.

$4x^2 - 9$

$a = 2x, b = 3$

$(2x + 3)(2x - 3)$

Step 4: Write the complete factorization.

$4(2x + 3)(2x - 3)$

Check:

$4(2x + 3)(2x - 3) = 4(4x^2 - 6x + 6x - 9)$

$= 4(4x^2 - 9)$

$= 16x^2 - 36$ ✓

Factor each polynomial completely.

4. $3x^2 - 300$ — $3(x + 10)(x - 10)$

5. $4x^2 - 20x - 24$ — $4(x + 1)(x - 6)$

6. $8x^2 - 40x + 50$ — $2(2x - 5)^2$

7. $-7x^2 - 21x + 28$ — $-7(x + 4)(x - 1)$

8. $8x^2 - 18$ — $2(2x + 3)(2x - 3)$

9. $20x^2 + 50x + 30$ — $10(2x + 3)(x + 1)$

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LESSON 8-6

Challenge

Sum and Difference of Cubes

A sum of cubes can be factored using: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Factor $x^3 + 125y^3$.

$(x)^3 + (5y)^3$ — $a = x, b = 5y$

$(x + 5y)(x^2 - 5xy + 25y^2)$ — *Substitute into* $(a + b)(a^2 - ab + b^2)$.

Factor each polynomial.

1. $x^3 + 8$ — $(x + 2)(x^2 - 2x + 4)$

2. $64x^3 + y^3$ — $(4x + y)(16x^2 - 4xy + y^2)$

3. $125x^3 + 216$ — $(5x + 6)(25x^2 - 30x + 36)$

A difference of cubes can be factored using: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Factor $8x^3 - 27$.

$(2x)^3 - (3)^3$ — $a = 2x, b = 3$

$(2x - 3)(4x^2 + 6x + 9)$ — *Substitute into* $(a - b)(a^2 + ab + b^2)$.

Factor each polynomial.

4. $x^3 - 64$ — $(x - 4)(x^2 + 4x + 16)$

5. $343x^3 - y^3$ — $(7x - y)(49x^2 + 7xy + y^2)$

6. $729x^3 - 1000$ — $(9x - 10)(81x^2 + 90x + 100)$

You can use the sum and difference of cubes with other factoring methods to completely factor a polynomial.

Factor each polynomial completely.

7. $2x^3 - 54$ — $2(x - 3)(x^2 + 3x + 9)$

8. $216x^3 + 8y^3$ — $8(3x + y)(9x^2 - 3xy + y^2)$

9. $x^6 + 12x^3 + 32$ — $(x + 2)(x^2 - 2x + 4)(x^3 + 4)$

10. $3x^6 + 18x^3 + 15$ — $3(x^3 + 5)(x + 1)(x^2 - x + 1)$

11. $x^5 - 27x^2 - 9x^3 + 243$ — $(x + 3)(x - 3)^2(x^2 + 3x + 9)$

Use difference of squares to factor each binomial. Then factor completely.

12. $729x^6 - 64$ — $(3x + 2)(3x - 2)(9x^2 - 6x + 4)(9x^2 + 6x + 4)$

13. $64x^6 - y^6$ — $(2x + y)(2x - y)(4x^2 - 2xy + y^2)(4x^2 + 2xy + y^2)$

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LESSON 8-6

Problem Solving

Choosing a Factoring Method

Write the correct answer.

1. A rectangular stage set up in a theater has an area of $(15x^2 + 3x - 12)$ square feet. Factor the polynomial completely.

$3(5x - 4)(x + 1)$

2. The area of a circular rug is $(16\pi k^2 - 16\pi k + 4\pi)$ m^2. Factor the expression completely. Then find the area of the rug if $k = 1$ meter.

$4\pi(2k - 1)^2$;

4π m^2

3. An artist framed a picture. The dimensions of the picture and frame are shown below.



Completely factor the expression for the area of the frame.

$4(2x + y)(2x - y)$

4. The attendance for a team's basketball game can be approximated with the polynomial $-5x^2 + 80x + 285$, where x is the number of wins the team had in the previous month. Factor the polynomial completely. Then predict the attendance when the team won 4 games in the previous month.

$-5(x - 19)(x + 3)$;

525 attendees

Select the best answer.

5. The volume of a box can be modeled by the expression $7x^4 - 28$. Which shows this expression completely factored?

A $7(x^4 - 4)$

B $7(x^2 - 2)^2$

C $(7x^2 + 4)(x^2 - 7)$

(D) $7(x^2 + 2)(x^2 - 2)$

6. The area of a Japanese rock garden is $(30x^2 + 3x - 6)$ square feet. Factor the polynomial completely.

F $3(10x^2 + x - 2)$

(G) $3(2x + 1)(5x - 2)$

H $(6x + 3)(5x - 2)$

J $(15x - 6)(2x + 1)$

7. The money made from the sales of x mountain bikes is approximated by $20x^2 + 10x + 90$. Factor the expression completely.

A $2(10x + 9)(x + 5)$

B $5(4x^2 + 2x + 18)$

(C) $10(2x^2 + x + 9)$

D The expression cannot be factored.

8. Kyle stood on a bridge and threw a rock up and over the side. The height of the rock, in meters, can be approximated by $-5t^2 + 5t + 24$, where t is the time in seconds after Kyle threw it. Completely factor the expression.

F $-5(t^2 + t + 24)$

G $(-5t + 3)(t + 8)$

H $-1(5t + 8)(t + 3)$

(J) The expression cannot be factored.

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LESSON 8-6

Reading Strategies

Use a Sequence Chain

Use the sequence chain below to guide you in factoring polynomials.

Factoring Polynomials

Is there a GCF among the terms?
If yes, factor it out.
If no, move on.

How many terms are in the polynomial?
If 2 If 3 If 4

Is the binomial a difference of squares $a^2 - b^2$?
If yes, factor it into $(a + b)(a - b)$.
If no, you're done.

Is this a perfect square trinomial?
If yes, factor it into $(a + b)^2$ or $(a - b)^2$.
If no,

Are there terms with common factors?
If yes, try factor by grouping.
If no, you're done.

Can this trinomial be factored into 2 binomials?
If yes, factor it.
If no, you're done.

Complete the following.

1. What is the first thing to look for when factoring any polynomial? GCF

2. If the polynomial has 2 terms, what special product should you look for?

difference of squares

3. Describe the first step in factoring the polynomial $16x^2 + 12x - 20$.

Factor 4 out.

Factor each polynomial using the sequence chain as a guide.

4. $2x^2 - 15x - 27$ — $(2x + 3)(x - 9)$

5. $3x^2 - 75$ — $3(x + 5)(x - 5)$

6. $18x^4 + 12x^2 + 2$ — $2(3x^2 + 1)^2$

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