SECTION 10A Ready to Go On? Quiz

10-1 Probability

Use the table to find the probability of each event.

Outcome	A	B	C	D
Probability	0.1	0.3	0.5	0.1

1. P(B) 0.3 **2.** P(A or B) 0.4 **3.** P(not C) 0.5 **4.** P(B, C, or D) 0.9

At an ice cream shop, customers can choose up to 6 different toppings to be blended with their ice cream. The table gives the probability of each outcome.

Outcome	0	1	2	3	4	5	6
Probability	0.013	0.142	0.361	0.254	0.136	0.083	0.011

5. What is the probability of someone getting less than 3 toppings? 0.516

6. What is the probability of someone getting 3 or 4 toppings? 0.39

People leaving a theater rated the movie they saw on a scale of 0 ("Hated it") to 6 ("Loved it"). The table gives the probability of each outcome.

Outcome	0	1	2	3	4	5	6
Probability	0.004	0.011	0.081	0.163	0.216	0.422	0.103

7. What is the probability that a rating is over 4? 0.525

8. What is the probability that a rating is *not* the highest or lowest possible? 0.893

9. Four students are finalists in the competition for best science essay. Kyle has a 45% chance of winning. Jim's chance of winning is one-third of Kyle's. Mia's chance is the same as Amy's. Create a table of probabilities for the sample space.

Outcome	Kyle	Jim	Mia	Amy
Probability	0.45	0.15	0.2	0.2

10. Five frogs are in a jumping contest. Luke has 20% chance of winning. Pip's chances are one-quarter of that. Croak, Jojo, and Max have equal chances. Create a table of probabilities for the sample space.

Outcome	Luke	Pip	Croak	Jojo	Max
Probability	0.20	0.05	0.25	0.25	0.25

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SECTION 10A Ready to Go On? Quiz continued

10-2 Experimental Probability

A colored chip is randomly drawn from a box and then replaced. The table shows the results after 300 draws.

Outcome	Red	Green	Blue	Yellow
Probability	78	57	141	24

11. Estimate the probability of drawing a blue chip. 0.47

12. Estimate the probability of drawing a red chip or a yellow chip. 0.34

13. Estimate the probability of drawing a chip that is not green. 0.81

10-3 Use a Simulation

Use the table of random numbers to simulate each situation. Use 10 trials for each simulation.

57245	39666	18545	50534	57654	25519	35477
42726	58321	59267	72742	53968	63679	54095
82768	32694	62828	19097	09877	32093	23518
97742	58918	33317	34192	06286	39824	74264
48332	38634	20510	09198	56256	04431	22753
26700	40484	28341	25428	08806	98858	04816

14. A survey shows that in Central City, 71% of the households have more than one television set. Estimate the probability that at least 5 out of 10 randomly chosen households have more than one television set. 0.80

15. A carton contains 100 pairs of socks. Of these, 33 pairs of socks are brown. If you randomly select 6 pairs of socks, what is the probability that at least 2 pairs will be white? 0.40

16. In a college town, 2 out of every 5 people is a college student. If you randomly select 7 people on the street, what is your estimate of the probability that at least 4 of those 7 people are college students? 0.20

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SECTION 10A Ready to Go On? Enrichment

Random Numbers and Codes

You have seen how random digits can be used to simulate random events. Random digits are also central to encryption, which means putting plain text into code.

The sender and receiver of a coded message have to use the same key. A simple key might be: Each code letter is 10 positions later in the alphabet than the text letter. Thus the text *We can meet at five in the park* would be encrypted as GO MKX WOOD KD PSFO SX DRO ZKBU.

1. Suppose the key is: Each code letter is 5 positions later in the alphabet than the text letter. Write the coded message for *I will call you tomorrow.*

N BNQQ HFQQ DTZ YTRTWWTB

2. If someone intercepted this message and suspected that the code involved a simple shift in the alphabet, how would they go about breaking the code?

Try a shift 1 letter ahead, then 2 letters ahead, and so on until they see real words being formed.

A more complicated code involves the use of what is called a one-time pad (OTP). Used only for one message, the OTP contains a sequence of random numbers. Every character in the message is encrypted with a different number. Suppose the sequence in the OTP consists of random numbers in the range 00 to 26. If the sequence begins 052501, that means the first letter in the text is 5 positions later in the alphabet than the first letter in the code, the second is 25 positions later, and the third is 1 position later. Thus, if the first word in the text is *Why*, the code would read BFZ.

3. Use the OTP on the right to encrypt the message *I will call you tomorrow.*

020720 011215 262120 171311 032612 092324 072019

K DBMX RZFE PBF WNYXNOVP

4. Compare the two encryptions you wrote for the same sentence. What clues about the original text does the simple code give someone that the OTP code does not?

You can see what letters are doubled. You know that N stands for either A or I. Since the W is doubled, it cannot be C, H, I, J, K, Q, U, V, W, X, Y, or Z.

5. Use the random number generator in a spreadsheet program to create your own OTP. Use the OTP to encode a message. Give the OTP and the coded message to a partner for decoding, and decode the message your partner gives you.

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LESSON 10-4 Ready to Go On? Skills Intervention

Theoretical Probability

Theoretical probability is used to estimate probabilities by making assumptions about an experiment.

Vocabulary
theoretical probability

$$\text{Probability of an event} = \frac{\text{number of equally likely outcomes in an event}}{\text{total number of possible outcomes}}$$

Calculating Theoretical Probability

An experiment consists of rolling a specially-numbered fair number cube. There are 6 possible outcomes: 2, 4, 6, 8, 10, and 12. What is the probability of rolling a 6?

What does a fair number cube mean? Each event is equally likely.

How many 6's could be rolled? 1

The probability of rolling a 6 is $P(6) = \frac{1}{6}$.

Calculating Probability for Two Fair Number Cubes

An experiment consists of rolling two fair number cubes.

A. What is the probability that the total shown on both number cubes is 4?

How many outcomes are possible? 3

$P(\text{total} = 4) = \frac{3}{36} = \frac{1}{12}$

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

B. What is the probability that the total shown on both number cubes is greater than 9?

List the outcomes that are possible: (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), and (6, 6).

$P(\text{total} > 9) = \frac{6}{36} = \frac{1}{6}$

Finding the Probability of Mutually Exclusive Events

Suppose you are playing a game in which two fair number cubes are rolled. You need a total of 9 to finish the game by an exact count, or 5 to land on a "free space." What is the probability of landing on finish or a "free space"? Use the table above.

The event "total = 9" consists of how many outcomes? 4

The event "rolling a 5" consists of how many outcomes? 4

$P(\text{win or free space}) = \frac{4}{36} + \frac{4}{36} = \frac{8}{36} = \frac{2}{9}$

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LESSON 10-9 Ready to Go On? Skills Intervention
Permutations and Combinations

A **permutation** is an arrangement of things in a particular order.

$${}_nP_r = \frac{n!}{(n-r)!}$$

A **combination** is a selection of things in any order.

$${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

A **factorial** is the product of all whole numbers from the given number down to 1.

Vocabulary
permutation
combination
factorial

Evaluating Expressions Containing Factorials

Evaluate each expression.

A. 6!

What are the whole numbers less than or equal to 6? 6, 5, 4, 3, 2, 1

Multiply. 6 • 5 • 4 • 3 • 2 • 1 = 720

B. $\frac{8!}{3!}$

Write the factorial of each number.

$$\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$

Cancel common factors.

Multiply the remaining factors.

8 • 7 • 6 • 5 • 4 = 6,720

Finding Permutations

There are 6 cross-country runners running a race. Find the number of orders in which all 6 runners can finish.

What is the formula for permutations?

$${}_nP_r = \frac{n!}{(n-r)!}$$

How many runners are in the race?

n = 6

How many runners are running at a time? r = 6

Substitute into the formula:

$${}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$$

= 720

There are 720 permutations. This means that there are 720 orders in which the 6 runners can finish running the race.

Finding Combinations

Student council needs to select a dance committee of 4 members from the group of 10 council members. Find the number of possible combinations.

Find the total number of council members. n = 10

How many are chosen at a time? r = 4

Substitute into the combination formula.

$${}_{10}C_4 = \frac{10!}{4!(10-4)!}$$

Write out the factorials. Cancel and multiply.

$${}_{10}C_4 = \frac{10!}{4!(6)!} =$$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{(4 \cdot 3 \cdot 2 \cdot 1)(\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})} = \frac{5040}{24}$$

= 210

There are 210 4-member committees that can be formed.

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LESSON 10-9 Ready to Go On? Problem Solving Intervention
Permutations and Combinations

When you calculate permutations and combinations, you may get results that surprise you.

Think of a standard deck of 52 playing cards. Are there more possible 13-card hands or 39-card hands? Explain.

Understand the Problem

1. Is a hand a permutation or combination? *Hint:* Does order matter?

combination

Make a Plan

2. Write an expression with factorials for the number of possible 13-card hands. *Hint:* Think of ${}_{52}C_{13}$.

$${}_{52}C_{13} = \frac{52!}{13!\,(52-13)!}$$

3. Write an expression with factorials for the number of possible 39-card hands.

$${}_{52}C_{39} = \frac{52!}{39!\,(52-39)!}$$

Solve

4. Without calculating the factorials, compare the two expressions you wrote.

$${}_{52}C_{13} = \frac{52!}{13!\,(52-13)!} = \frac{52!}{13!\,(39!)} \qquad {}_{52}C_{39} = \frac{52!}{39!\,(52-39)!} = \frac{52!}{39!\,(13!)}$$

They are equal.

5. Are there more possible 13-card hands or 39-card hands? Explain.

There are the same number of each because $\frac{52!}{13!\,(39!)} = \frac{52!}{39!\,(13!)}$.

Check

6. Why does your answer make sense? *Hint:* Think of what is left each time you deal out a 13-card hand from a deck.

for each 13-card hand you make, there are 39 cards left

Solve

7. There are 73 students to choose from. Which is greater, the number of possible teams of 5 or the number of possible teams of 68? Explain.

The same; for every combination of 5, there is a combination of 68

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SECTION 10B Ready to Go On? Quiz

10-4 Theoretical Probability

An experiment consists of rolling two fair number cubes. Find the probability of each event.

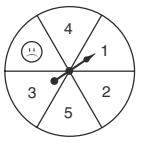
1. P(total shown = 5) $\frac{1}{9}$
2. P(a 2 and a 6) $\frac{1}{18}$
3. P(total less than 8) $\frac{7}{12}$

10-5 Independent and Dependent Events

4. Two fair number cubes are rolled. What is the probability that the first number rolled is a 4 and that the second number rolled is a 1? $\frac{1}{36}$
5. A set of 10 cards is numbered 1–10. If two cards are chosen at random, what is the probability that they are both even? $\frac{2}{9}$
6. A bag contains 3 red tokens, 6 green tokens, 2 yellow tokens, and 1 blue token. If two tokens are chosen at random, what is the probability that 1 is yellow and the other is not yellow? $\frac{20}{121}$

10-6 Making Decisions and Predictions

7. The spinner shown is used to tell a player how many cards to draw. Landing on the sad face means no cards get drawn. Suppose the spinner is spun 120 times. Predict how many times it will land on the sad face.



20 times

8. A set of 40 cards is numbered 1–40. Player A wins the round if the card drawn is even and under 21. Player B wins the round if the card drawn is odd and over 20. Decide whether the game is fair and show why.

fair; $\frac{1}{4} = \frac{1}{4}$

9. The table shows the sizes of the last 500 T-shirts sold. Kia is placing an order for 1,200 T-shirts. How many medium size T-shirts should she order?

T-shirts Sold		
Small	Medium	Large
119	225	156

540

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SECTION 10B Ready to Go On? Quiz continued

10-7 Odds

10. If the odds in favor of winning the grand prize in a drawing are 1:14,999, what is the probability of winning the grand prize? $\frac{1}{15{,}000}$
11. If you send in a letter to the *Ask Bess* TV show, the probability that Bess will read it aloud on her show is $\frac{1}{999}$. What are odds that your letter will *not* get read aloud on the show? 1,000:1

10-8 Counting Principles

12. The menu in a restaurant has a choice of 3 kinds of soup, 4 different main courses, and 5 different desserts. How many different orders can a waitress hear from someone who is ordering soup, a main course, and dessert? 60
13. How many 7-digit phone numbers are there that contain only the digits 1, 3, and 5? 2,187
14. Paul is making up password with 4 digits and 2 letters to use with his banking account on the Internet. How many choices of passwords does he have? 6,760,000

10-9 Permutations and Combinations

Evaluate each expression

15. 6! 720
16. $\frac{8!}{5!}$ 336
17. $\frac{7!}{(5-2!)}$ 840
18. There are 5 dogsleds in a race. In how many possible orders can all 5 dogsleds finish the race? 720
19. In a school board election, 8 candidates are running for 3 seats. How many different choices does Mrs. Lantera have if she is voting for 3 of the candidates? 56

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