LESSON 13-3

Ready to Go On? Skills Intervention

Other Sequences

Using First and Second Differences

Use first and second differences to find the next three terms in each sequence. Complete each table.

A. 3, 8, 15, 24, 35, 48, 63, …

Sequence	3	8	15	24	35	48	63	??	??	??
1st Differences		5	7	9	11	13	15	17	19	21
2nd Differences			2	2	2	2	2	2	2	2

The next three terms are 80, 99, and 120.

B. 2, 8, 18, 32, 50, 72, …

Sequence	2	8	18	32	50	72	??	??	??
1st Differences		6	10	14	18	22	26	30	34
2nd Differences			4	4	4	4	4	4	4

The next three terms are 98, 128, and 162.

Finding a Rule Given Terms of a Sequence

Give the next three terms in the sequence using the simplest rule you can find.

1, 8, 27, 64, 125, …

Are the terms perfect squares? no

Are the terms perfect cubes? yes

The next three terms are 216, 343, and 512.

Finding Terms of a Sequence Given a Rule

Find the first five terms of the sequence defined by $a_n = \frac{4n}{n+2}$.

What does the n stand for? the number of the term

$a_1 = \frac{4(1)}{1+2} = \frac{4}{3} \longrightarrow a_2 = \frac{4(2)}{2+2} = \frac{8}{4} = 2 \longrightarrow a_3 = \frac{4(3)}{3+2} = \frac{12}{5}$

$a_4 = \frac{4(4)}{4+2} = \frac{16}{6} = \frac{8}{3} \longrightarrow a_5 = \frac{4(5)}{5+2} = \frac{20}{7}$

The first five terms are $\frac{4}{3}$, 2, $\frac{12}{5}$, $\frac{8}{3}$, $\frac{20}{7}$.

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SECTION 13A

Ready to Go On? Quiz

13-1 Terms of Arithmetic Sequences

Determine whether each sequence could be arithmetic. If so, give the common difference.

1. 9, 10, 12, 13, ... no

2. 17, 34, 51, 68, ... yes; 17

3. 39, 52, 65, 78, ... yes; 13

4. 19, 31, 43, 55, ... yes; 12

5. 56, 49, 42, 35, ... yes; −7

6. 14, 51, 88, 125, ... yes; 37

Find the given term in each arithmetic sequence.

7. 7th term: 4, 13, 22, ... 58

8. 10th term: $6\frac{2}{3}$, $7\frac{1}{3}$, ... 12

9. 12th term: 4, 5.4, 6.8, ... 19.4

10. 8th term: 22, 19.5, 17, ... 4.5

11. Henriqueta earns $9.00 the first hour and $6.50 every hour after that. How much money has she earned after 8 hours? $54.50

13-2 Terms of Geometric Sequences

Determine whether each sequence could be geometric. If so, give the common ratio.

12. 3, −6, 12, −24, ... yes; −2

13. 2, 4, −8, 16, ... no

14. −1, 5, −25, 125, ... yes; −5

15. 18, 9, 4.5, 2.25, ... yes; 0.5

16. 4, 16, 64, 256, ... yes; 4

17. 17, $8\frac{1}{2}$, $4\frac{1}{4}$, $2\frac{1}{8}$, ... yes; 0.5

Find the given term in each geometric sequence.

18. 6th term: 9, 18, 36, ... 288

19. 7th term: –4, 12, –36, ... −2,916

20. 5th term: –3, –15, –75, ... −1,875

21. 5th term: 90, –30, 10, ... $1\frac{1}{9}$

22. Francesco buys $100.00 of stock in a fast-growth company. It grows by 50% each year. To the nearest penny, what is Francesco's stock worth after 5 years? $759.38

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SECTION 13A

Ready to Go On? Quiz continued

13-3 Other Sequences

Use the first and second differences to find the next three terms in each sequence.

23. 3, 3, 4, 6, 9, ... 13, 18, 24

24. 2, 11, 22, 35, 50, ... 67, 86, 107

25. 13, 9, 6, 4, 3, ... 3, 4, 6

26. 4, 6, 6, 4, 0, ... −6, −14, −24

27. −74, −49, −29, −14, −4, ... 1, 1, −4

28. 1, 3, 6, 10, 15, … 21, 28, 36

Give the next three terms in each sequence using the simplest rule you can find.

29. 1, 2, 4, 8, 16, ... 32, 64, 128

30. 63, 31, 15, 7, 3, ... 1, 0, $-\frac{1}{2}$

31. $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{11}$, ... $\frac{12}{13}, \frac{14}{15}, \frac{16}{17}$

32. 243, 81, 27, 9, 3, ... 1, $\frac{1}{3}$, $\frac{1}{9}$

33. 2, 4, 10, 28, 82, ... 244; 730; 2,188

34. 240, 120, 60, 30, … 15, $7\frac{1}{2}$, $3\frac{3}{4}$

Find the first five terms of each sequence as defined by the given rule.

35. $a_n = 3n - 5$ −2, 1, 4, 7, 10

36. $a_n = -4n + 1$ −3, −7, −11, −15, −19

37. $a_n = (-1)^n$ −1, 1, −1, 1, −1

38. $a_n = (n - 3)^3$ −8, −1, 0, 1, 8

39. $a_n = (n + 1)^2 + 10$ 14, 19, 26, 35, 46

40. $a_n = 6n - 6$ 0, 6, 12, 18, 24

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SECTION 13A

Ready to Go On? Enrichment

Hidden Sequences

Find all the number sequences hidden in this array. Each sequence must have 4, 5, or 6 numbers. These must be arranged horizontally, vertically, or diagonally in a straight line without gaps. Ring each sequence that you find. A number may belong to more than one sequence.

10.2	7	5	4	4	5	7	$\frac{3}{2}$	$23\frac{1}{2}$
11	8.5	0	−53	−23	2	22	37	47
$-1\frac{1}{4}$	10	6.8	6.2	$1\frac{1}{3}$	14	1	101	94
$2\frac{1}{2}$	11.5	1	5.1	8	1	225	$\frac{1}{2}$	188
−5	−2	1	4	7	45	$\frac{2}{3}$	$3\frac{1}{7}$	$\frac{1}{4}$
10	−0.5	2	2.9	9	3	1	$\frac{1}{3}$	4
−20	2	4	1.8	116	16	6	5.9	5.89
40	0.1	7	8.5	33	29	25	21	$-2\frac{1}{4}$
7	9	11	13	13	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$

How many sequences did you find? Answers will vary.

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LESSON 13-6

Ready to Go On? Skills Intervention

Quadratic Functions

A **quadratic function** is of the form: $f(x) = ax^2 + bx + c$.

The graph of a quadratic function is a **parabola.**

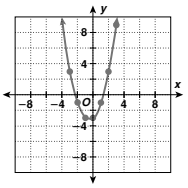
Vocabulary
quadratic function
parabola

Graphing Quadratic Functions
Create a table for the quadratic function and use it to make a graph.

$f(x) = x^2 + x - 3$

Plot the points and connect with a smooth curve.

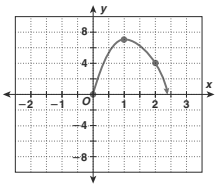
x	$f(x) = x^2 + x - 3$		
−3	$f(x) = (-3)^2 + (-3) - 3$	$= 9 - 6$	$= 3$
−2	$f(x) = (\underline{-2})^2 + (-2) - 3$	$= 4 - \underline{5}$	$= \underline{-1}$
−1	$f(x) = (\underline{-1})^2 + (-1) - 3$	$= 1 - \underline{4}$	$= \underline{-3}$
0	$f(x) = (\underline{0})^2 + 0 - \underline{3}$	$= \underline{0} - \underline{3}$	$= \underline{-3}$
1	$f(x) = (\underline{1})^2 + 1 - 3$	$= \underline{1} - \underline{2}$	$= \underline{-1}$
2	$f(x) = (\underline{2})^2 + \underline{2} - \underline{3}$	$= \underline{4} - \underline{1}$	$= \underline{3}$
3	$f(x) = (3)^2 + 3 - 3$	$= 9 - 0$	$= 9$



Sports Application
The height in meters of a ball tossed into the air after s seconds is given by the function $-5s^2 + 12s$. Graph the function and find the height the ball reaches after 2 seconds.

First, create a table of values.

s	$f(s)$
0	$-5(0)^2 + 12(0) = \underline{0}$
1	$-5(1)^2 + 12(\underline{1}) = \underline{7}$
2	$-5(\underline{2})^2 + 12(\underline{2}) = \underline{4}$



Why can s not be negative?

s represents time and time can't be negative.

Why is $f(2)$ less than $f(1)$? The ball has started falling.

What is the highest point the ball reaches? 7 m

What height does it reach after 2 seconds? 4 m

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LESSON 13-6

Ready to Go On? Problem Solving Intervention

Quadratic Functions

Find two ordered pairs that are solutions to both $y = 3x + 2$ and $y = x^2 + x - 6$.

Understand the Problem

1. What should happen when you substitute your answers into the equations $y = 3x + 2$ and $y = x^2 + x - 6$?

Both equations should be true.

Make a Plan

2. How would a sketch of the graphs of the two equations help you estimate the answer?

The points where the graphs meet are solutions to both equations.

Solve

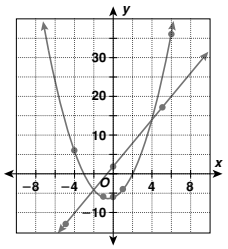
3. Complete the function table for each equation. Then sketch the graphs.

$y = 3x + 2$

x	y
−5	−13
0	2
5	17

$y = x^2 + x - 6$

x	y
−4	6
−1	−6
0	−6
1	−4
6	36



4. Estimate the coordinates of the points where the graphs meet. Test your estimates. If they don't work, try nearby points. Which ordered pairs satisfy both equations?

(−2, −4) and (4, 14)

Check

5. Substitute your answers again into both equations.

$(-4) = 3(-2) + 2,\ -4 = -4;\ (-4) = (-2)^2 + (-2) - 6,\ -4 = -4$

$(14) = 3(4) + 2,\ 14 = 14;\ (14) = (4)^2 + (4) - 6,\ 14 = 14$

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LESSON 13-7

Ready to Go On? Skills Intervention

Inverse Variations

In an **inverse variation** one variable increases in quantity while another variable decreases in quantity. The general form for an inverse variation is

$xy = k$ or $y = \frac{k}{x}$.

Vocabulary
inverse variation

Identifying Inverse Variation
Tell whether the relationship is an inverse variation.

A car is traveling a distance of 20 miles. The table shows the length of time it will take to travel 20 miles at a particular speed.

Speed (mi/h)	5	20	40	50
Time (h)	4	1	0.5	0.4

Find each product:

$5(4) = \underline{20}$ $(20)(1) = \underline{20}$ $(40)(\underline{0.5}) = \underline{20}$ $(50)(\underline{0.4}) = \underline{20}$

Is the product of xy the same for each set of numbers? yes
Is the relationship an inverse variation? yes

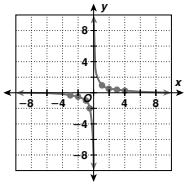
Write the inverse variation: $y = \frac{20}{x}$.

Graphing Inverse Variations
Graph the inverse variation function.

$f(x) = \frac{1}{2x}$

Create a table of values for the function. The graph will be in quadrants I and III.

x	$f(x) = \frac{1}{2x}$	x	$f(x) = \frac{1}{2x}$
−3	$f(x) = \frac{1}{2(-3)} = \frac{-1}{6}$	1	$f(x) = \frac{1}{2(1)} = \frac{1}{2}$
−2	$f(x) = \frac{1}{2(-2)} = \frac{-1}{4}$	2	$f(x) = \frac{1}{2(2)} = \frac{1}{4}$
−1	$f(x) = \frac{1}{2(-1)} = \frac{-1}{2}$	3	$f(x) = \frac{1}{2(3)} = \frac{1}{6}$
$-\frac{1}{2}$	$f(x) = \frac{1}{2\left(\frac{-1}{2}\right)} = -1$	4	$f(x) = \frac{1}{2(4)} = \frac{1}{8}$



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SECTION 13B

Ready to Go On? Quiz

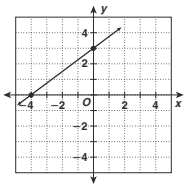
13-4 Linear Functions

Determine whether each function is linear.

1. $f(x) = \frac{4}{7}x - \frac{1}{7}$ yes
2. $f(x) = 1.85x + 9$ yes
3. $f(x) = 2x^4 + 1$ no

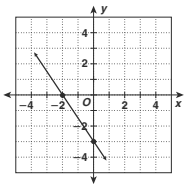
Write a rule for each function.

4.



$y = \frac{3}{4}x + 3$

5.



$y = -\frac{3}{2}x - 3$

6. Angela has a base salary of $220 per week. She also earns a commission of $20 on every widget she sells. Find a rule for the linear function that describes Angela's weekly income if she sells x widgets. How much does Angela earn if she sells 8 widgets in one week?

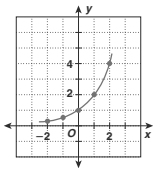
$f(x) = 20x + 220$; $380

13-5 Exponential Functions

Create a table for each exponential function and use it to graph the function.

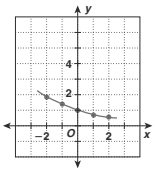
7. $f(x) = 2^x$

x	$f(x)$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2
2	4



8. $f(x) = \left(\frac{3}{4}\right)^x$

x	$f(x)$
−2	1.78
−1	1.33
0	1
1	0.75
2	0.56



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SECTION 13B

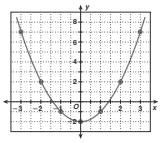
Ready to Go On? Quiz continued

13-6 Quadratic Functions

Create a table for each quadratic function, and use it to graph the function.

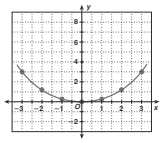
9. $f(x) = x^2 - 2$

x	$f(x)$
−3	7
−2	2
−1	−1
0	−2
1	−1
2	2
3	7



10. $f(x) = \frac{1}{3}x^2$

x	$f(x)$
−3	3
−2	$1\frac{1}{3}$
−1	$\frac{1}{3}$
0	0
1	$\frac{1}{3}$
2	$1\frac{1}{3}$
3	3



11. The function $f(x) = -16x^2 + 100x$ gives the height in feet after x seconds of an object shot upward from the ground at an initial velocity of 100 feet per second. After how many seconds is this height the greatest, 1, 2, 3, 4, or 5? What is this height?

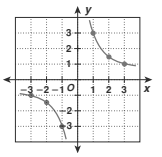
3 sec; 156 ft

13-7 Inverse Functions

Create a table. Then graph each inverse-variation function.

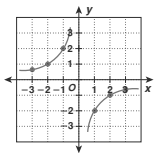
12. $f(x) = \frac{3}{x}$

x	$f(x)$
−3	−1
−2	−1.5
−1	−3
0	–
1	3
2	1.5
3	1



13. $f(x) = -\frac{2}{x}$

x	$f(x)$
−3	$\frac{2}{3}$
−2	1
−1	2
0	–
1	−2
2	−1
3	$-\frac{2}{3}$



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SECTION 13B

Ready to Go On? Enrichment

What You Are and What You Are Not

What are you about as a person? This activity will try to answer that question for you.

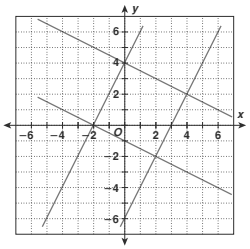
First, graph each of these four linear functions.

1. $f(x) = 2x + 4$

2. $f(x) = -\frac{1}{2}x + 4$

3. $f(x) = 2x - 6$

4. $f(x) = -\frac{1}{2}x - 1$



What figure is in the center? square

This figure is telling you what you are not.

Now graph each of these five linear functions.

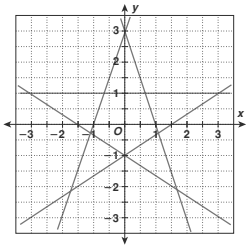
5. $f(x) = 3x + 3$

6. $f(x) = -3x + 3$

7. $f(x) = -\frac{2}{3}x - 1$

8. $f(x) = \frac{2}{3}x - 1$

9. $f(x) = 1$



What figure is formed by the graphs you draw? star

This figure is telling you what you are.

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LESSON 14-1

Ready to Go On? Skills Intervention

Polynomials

A **monomial** is a number or a product of numbers and variables with exponents that are whole numbers. A **polynomial** is one monomial or the sum or difference of polynomials. Polynomials can be classified by the number of terms. A monomial has 1 term, a **binomial** has 2 terms, and a **trinomial** has 3 terms. A polynomial can also be classified by its degree. The **degree of a polynomial** is the degree of the term with the greatest degree.

Vocabulary
- monomial
- polynomial
- binomial
- trinomial
- degree of a polynomial

Identifying Monomials

Determine whether each expression is a monomial.

A. $5x^{2.3}y$

Is this expression a product of numbers and variables? yes

Are all the exponents whole numbers? Explain.

No, the exponent 2.3 is not a whole number.

The expression is not a monomial.

B. $\frac{1}{3}x^4y^2$

Is this expression a product of numbers and variables? yes

Are all the exponents whole numbers? Explain.

Yes, both exponents are whole numbers.

The expression is a monomial.

Classifying Polynomials by the Number of Terms

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

A. $20.42t - 15.73r$

How many terms in this expression? 2

Are all the terms monomials? yes

The expression is a binomial.

B. $10gh - 4g + 5h$

How many terms in this expression? 3

Are all the terms monomials? Explain.

yes, all exponents are whole numbers

The expression is a trinomial.

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LESSON 14-1

Ready to Go On? Problem Solving Intervention

Polynomials

You can use polynomials to describe projectile motion and to find the height of an object at any given time.

The height in feet of a toy rocket launched straight up into the air from s feet above the ground at velocity v after t seconds is given by the polynomial $-16t^2 + vt + s$. Find the height of a toy rocket launched from a 2 ft platform at 160 ft/s after 5 seconds.

Understand the Problem

1. What does the polynomial in the problem represent?

the height of the toy rocket after t seconds

2. You have been given a polynomial with several variables. Two of them will be constant regardless of how much time passes. Which two variables will not change as time passes?

v and s

Make a Plan

3. Rewrite the polynomial expression for the height of the toy rocket by substituting the values for v and s.

$-16t^2 + 160t + 2$

4. Substitute $t = 0$ into the polynomial to find the height before the toy rocket is launched. Is the result consistent with the information given in the problem? Why or why not?

$-16(0)^2 + 160(0) + 2 = 2$ feet; Yes, since the platform is 2 feet tall.

Solve

5. The problem asks for the height after 5 seconds, so substitute $t = 5$ into the polynomial and simplify.

$-16(5)^2 + 160(5) + 2 = 402$ feet

6. Write your answer as a complete sentence. Be sure to include the appropriate units in your answer.

The height of the toy rocket after 5 seconds is 402 feet.

Check

7. Make sure your answer is reasonable by substituting another value of t close to 5, such as $t = 4$.

$-16(4)^2 + 160(4) + 2 = 386$ feet; 402 feet is reasonable for $t = 5$.

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