LESSON 14-2 Ready to Go On? Skills Intervention
Simplifying Polynomials

You can simplify a polynomial by adding or subtracting like terms. Remember that like terms have the same variables raised to the same powers.

Identifying Like Terms

Identify the like terms in each polynomial, or state that there are none.

A. $4b^2 - 2b + 2 + 3b^2 + 4b$

Which term(s) contain only b^2 as a variable? $4b^2$ and $3b^2$

Which term(s) contain only b as a variable? $-2b$ and $4b$

Which are like terms, if any? $4b^2$ and $3b^2$, $-2b$ and $4b$

B. $6g^2 + 5gh - 2g$

Which term(s) contain only g^2 as a variable? $6g^2$

Which term(s) contain only gh as a variable? $5gh$

Which term(s) contain only g as a variable? $2g$

Which are like terms, if any? There are no like terms.

C. $5x^2y^5 - 7y^3 - 2x^2y^5 + 3x^2y^5$

Which term(s) contain only x^2y^5 as the variables? $5x^2y^5$, $-2x^2y^5$, and $3x^2y^5$

Which term(s) contain only y^3 as a variable? $-7y^3$

What are the like terms, if any? $5x^2y^5$, $-2x^2y^5$, and $3x^2y^5$

Simplifying Polynomials by Combining Like Terms

Simplify.

A. $4y^4 - 2y^2 - 2y^4 + 3 + 5y^4 - y^2$

Arrange the terms in descending order. $4y^4 - 2y^4 + 5y^4 - 2y^2 - y^2 + 3$

Name the terms that have the same variable raised to the same power.

$4y^4$, $-2y^4$, $5y^4$; $-2y^2$ and $-y^2$

Are these like terms? yes

What term results from combining these terms? $7y^4$ and $-3y^2$

What is the polynomial after combining *all* the like terms? $7y^4 - 3y^2 + 3$

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LESSON 14-2 Ready to Go On? Problem Solving Intervention
Simplifying Polynomials

Polynomials are often used to represent growth and decay of physical elements.

A scientist is studying an element that he has discovered. He has determined that the amount of the element at any given time can be approximated by the polynomial $-0.25(4t^2 + At) + A$, where A is the original amount of the element and t is the number of hours that have passed. If the scientist originally had 1,000 grams of the element, how much is left after 2 hours?

Understand the Problem

1. What are the two variables in the problem?

   A and t

2. What is the value of A in the problem? Will it change for different values of t?

   1,000 grams; no

Make a Plan

3. Simplify the expression by substituting $A = 1{,}000$ and by multiplying the appropriate terms by -0.25.

   $-0.25(4t^2) + -0.25(1{,}000t) + 1{,}000 = -t^2 - 250t + 1{,}000$

4. What value of t will you need to substitute into the expression?

   $t = 2$

Solve

5. The problem asks for the amount of the element remaining after 2 hours, so substitute $t = 2$ into the polynomial and simplify.

   $-(2)^2 - 250(2) + 1{,}000 = 496$ grams

6. Write your answer as a complete sentence. Be sure to include the appropriate units in your answer.

   There are 496 grams of the element remaining after 2 hours have passed.

Check

7. Make sure your answer is reasonable by substituting another value of t close to 2, such as $t = 3$.

   $-(3)^2 - 250(3) + 1{,}000 = 241$ grams; Since the amount should be decreasing as time passes, 496 grams is reasonable for $t = 2$.

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SECTION 14A Ready to Go On? Quiz

14-1 Polynomials

Determine whether each expression is a monomial.

1. $\frac{1}{4}x^3$ — yes
2. $5a^7$ — yes
3. $2n^4 + m$ — no
4. $\frac{3}{cd}$ — no
5. $\frac{p^2}{6}$ — yes
6. $9r^2 + 1$ — no

Classify each expression as a monomial, a binomial, a trinomial, or not a polynomial.

7. $x^2 + 2x - \frac{4}{x}$ — not a polynomial
8. $3mnp^5$ — monomial
9. $y - 6\sqrt{y}$ — not a polynomial
10. $z^2 + 5 - 2z^3$ — trinomial
11. $4b - 7$ — binomial
12. $13abc^2pq^5$ — monomial
13. $4m^{3/2}$ — not a polynomial
14. $(x + 5)(x - 5)$ — binomial
15. $15r^3 - 2rs$ — binomial

Find the degree of each polynomial.

16. $4x^2 - 3x + 6$ — 2
17. $c^5 + 4c^3 - 11$ — 5
18. $9v^4 + 1$ — 4
19. $-5 + 3q^6 + q^2$ — 6
20. $b^2 + b - 2b^3$ — 3
21. $m - 3 + 2m^4 - 8m^3$ — 4
22. $12s^2$ — 2
23. $-1 - 17z^3$ — 3
24. $2w^3 + w^4$ — 4

25. The polynomial $2s + \frac{s^2}{20}$ gives an estimate in feet of how far a car going s mi/h must travel to come to a complete stop. Estimate this distance for a car going 30 mi/h. — 105 ft

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SECTION 14A Ready to Go On? Quiz continued

14-2 Simplifying Polynomials

Identify the like terms in each polynomial.

26. $-xy^2 + x^2y + 5xy^2$ — $-xy^2$, $5xy^2$
27. $p - 2p^3 + p^3 + 4p$ — $-2p^3$, p^3; p, $4p$
28. $wxy + 3wxy - 2wxy + xyz$ — wxy, $3wxy$, $-2wxy$
29. $-abc^2 + ab + bc^2 - 6abc^2$ — $-abc^2$, $-6abc^2$
30. $2m + 11n - m + m^4 + 3n$ — $2m$, $-m$; $11n$, $3n$
31. $-9p^2q + p^2q + 5p^2q - 2p^2q$ — $-9p^2q$, p^2q, $5p^2q$, $-2p^2q$
32. $4v - w + wx + 4v$ — $4v$, $4v$
33. $ab^2 - ab^2 + c^2d - 7c^2d + 2c^2d$ — ab^2, $-ab^2$; c^2d, $-7c^2d$, $2c^2d$

Simplify.

34. $4r + 10s - 3r + r^4 + 3s$ — $r + r^4 + 13s$
35. $x^2y + 7w - 2x^2y + z^4 + 8x^2y$ — $7x^2y + 7w + z^4$
36. $-pq - 3pq - pq^5 + 6pq$ — $2pq - pq^5$
37. $2(a^2 + 5a^2) - 20a^2 + 7(4a^2 - 2a^2)$ — $6a^2$
38. $6d^2 + 3(t - 2d^2)$ — $3t$
39. $m + 4n - p^2 - m - 5q$ — $4n - p^2 - 5q$
40. $ab^2 - ab^2 + c^2d - 2(7c^2d - 5c^2d)$ — $-3c^2d$
41. $14 - z^2 + 3u + 9z^2 + 100$ — $8z^2 + 3u + 114$

Solve.

42. One side of a regular pentagon is represented by the expression $3n - 2$. Write a polynomial to represent the perimeter of this pentagon. — $15n - 10$
43. An apartment building is a rectangular prism with a square base. The area of each side of the building is given by the expression $3a^2 + 5a - 2$. Write a polynomial to represent the total area of all 4 sides of the building. — $12a^2 + 20a - 8$

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LESSON 14-6

Ready to Go On? Problem Solving Intervention

Multiplying Binomials

Finding volume also involves multiplying expressions.

The base of a metal box is formed by cutting a 2-inch square out of each corner of a sheet of metal and folding up the sides. Write and simplify an expression for the volume of the box if the length of the sheet of metal is x inches, the width of the metal is y inches, and the height of the box is 2 inches.

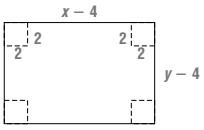
Understand the Problem

1. List the different variables that you will be using to answer the question. Then write a general formula for finding the volume of a rectangular box. Which of the three quantities is a constant in this problem?

x and y; $V = \ell wh$; $h = 2$ inches

Make a Plan

2. Begin by drawing a diagram. Label the diagram using the dimensions (and variables) given in the problem.



3. Use your diagram to rewrite the formula for volume using x and y.

$V = (x - 4)(y - 4)(2)$

Solve

4. You will need to multiply the two binomials and the number 2. Begin by multiplying the binomials.

$(x - 4)(y - 4) = xy - 4x - 4y + 16$

5. Next, multiply each of the terms by 2 to find the expression for the volume of the box.

$2xy - 8x - 8y + 32$

Check

6. Check your answer by choosing a value for x and y. Substitute your choices into the equation you wrote in Exercise 3 and into the simplified expression you wrote in Exercise 5. If you multiplied correctly, you should get the same result.

Possible answer: If $x = 5$ and $y = 6$, $(5 - 4)(6 - 4)(2) = (1)(2)(2) = 4$ and $2(5)(6) - 8(5) - 8(6) + 32 = 4$.

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SECTION 14B

Ready to Go On? Quiz

14-3 Adding Polynomials

Add.

1. $(2x^2 + 5x - 4) + (x^3 - 5x + 3)$ $x^3 + 2x^2 - 1$

2. $(2a^4 + a - 6) + (a^4 - 3a + 10)$ $3a^4 - 2a + 4$

3. $(4m^5n + mn^2 - 5) + (-2m^5n + 3mn^2 + 11) + (m^5n + 5mn^2 - 2)$

$3m^5n + 9mn^2 + 4$

4. $(7v^2 - 9v - 1) + (2v^2 + 2v + 11) + (-3v^2 + v - 7)$ $6v^2 - 6v + 3$

5. $(3w^3 + 3w^2 - 4) + (w^3 - 2w + 5) + (-w^3 + w^2)$ $3w^3 + 4w^2 - 2w + 1$

6. Each side of a regular pentagon has a length of $2d + 1$. Each side of a regular hexagon has a length of $3d - 7$. Write an expression for the sum of the two perimeters. $5(2d + 1) + 6(3d - 7)$

14-4 Subtracting Polynomials

Find the opposite of each polynomial.

7. $-y^2 + 4y + 5$ $y^2 - 4y - 5$

8. $2n^5 - 6n$ $-2n^5 + 6n$

9. $-7a^2b - a^2 + 3c$ $7a^2b + a^2 - 3c$

10. $pqr^3 + 5pqr^2 + pqr + q^4$ $-pqr^3 - 5pqr^2 - pqr - q^4$

Subtract.

11. $(3z^2 - z) - (5z^2 + 6z - 1)$ $-2z^2 - 7z + 1$

12. $(a + 9) - (4a^2 - 2)$ $-4a^2 + a + 11$

13. $(12p^3 + 5p) - (7p^3 + 6p)$ $5p^3 - p$

14. $(4t^2 - 3t + 6) - (-2t^2 + t + 7)$ $6t^2 - 4t - 1$

15. One solid figure has a volume of $12u^3 - u^2 + 4u + 7$. A smaller solid figure has a volume of $5u^3 + 5u^2 - 5u - 4$. Write an expression to show the difference between these two volumes.

$(12u^3 - u^2 + 4u + 7) - (5u^3 + 5u^2 - 5u - 4)$

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SECTION 14B

Ready to Go On? Quiz continued

14-5 Multiplying Polynomials and Monomials

Multiply.

16. $(-3a^2b^2)(2ab^3)$ $-6a^3b^5$

17. $(hj^2)(5h^5)$ $5h^6j^2$

18. $(-4p^7q)(-3p^4q^3)$ $12p^{11}q^4$

19. $(3x^8y^2)(-3y^2)$ $-9x^8y^4$

20. $(-r^6s^2)(5rs)$ $-5r^7s^3$

21. $(4n^3z^2)(5nz)$ $20n^4z^3$

22. $2u^2v(2u^3v - uv^2 + 4uv)$ $4u^5v^2 - 2u^3v^3 + 8u^3v^2$

23. $-3st^4(-2s^2t + st^2 + 4st^3)$ $6s^3t^5 - 3s^2t^6 - 12s^2t^7$

24. A parallelogram has a base of $4xy$ and a height of $3x^3 + 2xy^2 - y^2$. Write and then simplify an expression for the area of this parallelogram. Last, find the area of the parallelogram if $x = 1$ and $y = 2$.

$4xy(3x^3 + 2xy^2 - y^2)$; $12x^4y + 8x^2y^3 - 4xy^3$; 56 units^2

14-6 Multiplying Binomials

Multiply.

25. $(m - 3)(m + 7)$ $m^2 + 4m - 21$

26. $(2j - 4)(j - 5)$ $2j^2 - 14j + 20$

27. $(g + 9)(g + 6)$ $g^2 + 15g + 54$

28. $(3z + 1)(2z - 2)$ $6z^2 - 4z - 2$

29. $(2q - 3)(2q - 4)$ $4q^2 - 14q + 12$

30. $(4a - 1)(2a + 3)$ $8a^2 + 10a - 3$

31. A swimming pool is built in a 30 m • 20 m area so that there is a walkway x m wide all the way around the pool. Find the area of the pool in terms of x. What would the area of the pool be if x were 2?

$(30 - 2x)(20 - 2x)$ or $600 - 100x + 4x^2$; 416 m^2

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SECTION 14B

Ready to Go On? Enrichment

Multiplying Binomials and Factoring Polynomials

Multiply.

A. $(x + 2)(2x - 1)$ $2x^2 + 3x - 2$

B. $(2x - 3)(2x + 3)$ $4x^2 - 9$

C. $(3x + 1)^2$ $9x^2 + 6x + 1$

D. $(3x + 2)(4x - 3)$ $12x^2 - x - 6$

E. $(2x + 1)(x - 2)$ $2x^2 - 3x - 2$

F. $(x - 1)(5x - 2)$ $5x^2 - 7x + 2$

G. $(4x - 1)(2x + 3)$ $8x^2 + 10x - 3$

H. $(3x - 2)(3x + 2)$ $9x^2 - 4$

Now use the exercises you have just completed, together with your answers, to find the binomial factors of each polynomial below.

1. $9x^2 + 6x + 1$ $(3x + 1)^2$

2. $9x^2 - 4$ $(3x - 2)(3x + 2)$

3. $5x^2 - 7x + 2$ $(x - 1)(5x - 2)$

4. $2x^2 + 3x - 2$ $(x + 2)(2x - 1)$

5. $12x^2 - x - 6$ $(3x + 2)(4x - 3)$

6. $8x^2 + 10x - 3$ $(4x - 1)(2x + 3)$

7. $4x^2 - 9$ $(2x - 3)(2x + 3)$

8. $2x^2 - 3x - 2$ $(2x + 1)(x - 2)$

You know how to multiply two binomials to get a polynomial product. Explain why it is possible to go the other way and factor a polynomial into binomial factors.

Since you can multiply binomials to make a polynomial, some polynomials must have binomials as factors.

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