DIMENSIONAL ANALYSIS AND ORDER-OF-MAGNITUDE ESTIMATES

Note: In developing answers to order-of-magnitude calculations, you should state your important assumptions, including the numerical values assigned to parameters used in the solution. Since only order-of-magnitude results are expected, do not be surprised if your results differ from those of other students.

Review Questions

23. Suppose that two quantities, A and B, have different dimensions. Which of the following arithmetic operations *could* be physically meaningful?

a. $A + B$
b. A/B
c. $A \times B$
d. $A - B$

24. Estimate the order of magnitude of the length in meters of each of the following:

a. a ladybug
b. your leg
c. your school building
d. a giraffe
e. a city block

25. If an equation is dimensionally correct, does this mean that the equation is true?

26. The radius of a circle inscribed in any triangle whose sides are a, b, and c is given by the following equation, in which s is an abbreviation for $(a + b + c) \div 2$. Check this formula for dimensional consistency.

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

27. The period of a simple pendulum, defined as the time necessary for one complete oscillation, is measured in time units and is given by the equation

$$T = 2\pi\sqrt{\frac{L}{a_g}}$$

where L is the length of the pendulum and a_g is the acceleration due to gravity, which has units of length divided by time squared. Check this equation for dimensional consistency.

Conceptual Questions

28. In a desperate attempt to come up with an equation to solve a problem during an examination, a student tries the following: (velocity in m/s)^2 = (acceleration in m/s^2) $\times$ (time in s). Use dimensional analysis to determine whether this equation might be valid.

29. Estimate the number of breaths taken by a person during 70 years.

30. Estimate the number of times your heart beats in an average day.

31. Estimate the magnitude of your age, as measured in units of seconds.

32. An automobile tire is rated to last for 50 000 mi. Estimate the number of revolutions the tire will make in its lifetime.

33. Imagine that you are the equipment manager of a professional baseball team. One of your jobs is to keep a supply of baseballs for games in your home ballpark. Balls are sometimes lost when players hit them into the stands as either home runs or foul balls. Estimate how many baseballs you have to buy per season in order to make up for such losses. Assume your team plays an 81-game home schedule in a season.

34. A chain of hamburger restaurants advertises that it has sold more than 50 billion hamburgers over the years. Estimate how many pounds of hamburger meat must have been used by the restaurant chain to make 50 billion hamburgers and how many head of cattle were required to furnish the meat for these hamburgers.

35. Estimate the number of piano tuners living in New York City. (The population of New York City is approximately 8 million.) This problem was first proposed by the physicist Enrico Fermi, who was well known for his ability to quickly make order-of-magnitude calculations.

36. Estimate the number of table-tennis balls that would fit (without being crushed) into a room that is 4 m long, 4 m wide, and 3 m high. Assume that the diameter of a ball is 3.8 cm.

23. b, c
24. **a.** 10^{-2} m
b. 10^{0} m
c. 10^{1} m to 10^{2} m
d. 10^{1} m
e. 10^{2} m
25. No, dimensional consistency indicates that the equation *may* be valid, but it is not *necessarily* valid.
26. The dimensions are consistent.
27. The dimensions are consistent.
28. The equation is not valid.

(Note: Because the following are estimates, student answers may vary from those shown here.)
29. 4×10^{8} breaths
30. 9×10^{4} beats
31. 5.4×10^{8} s
32. 4×10^{7} revolutions
33. 2×10^{3} balls
34. 1×10^{10} lb, 2×10^{7} head of cattle
35. 7×10^{2} tuners
36. 8×10^{5} balls